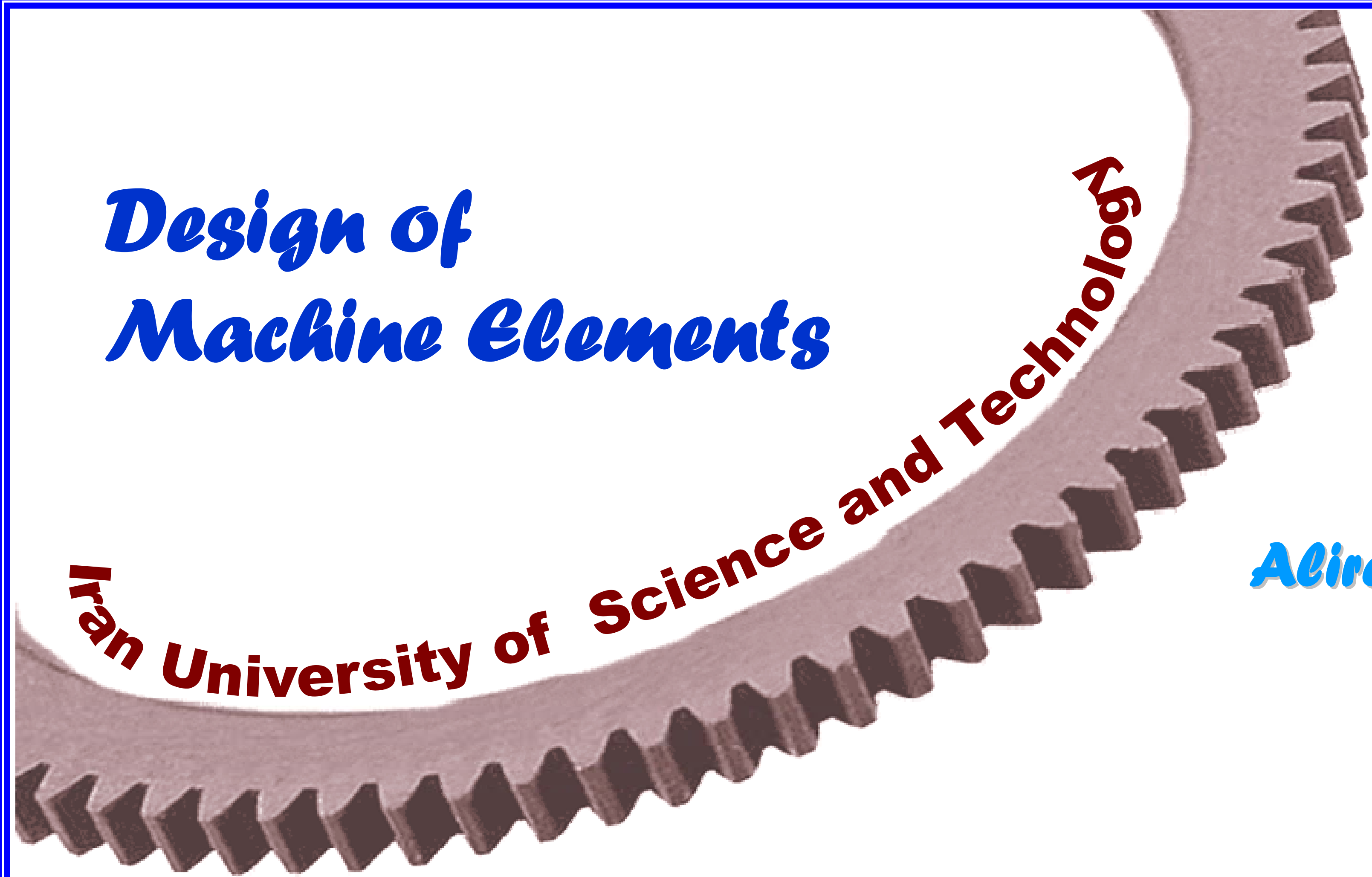
Design of Machine Elements



By:

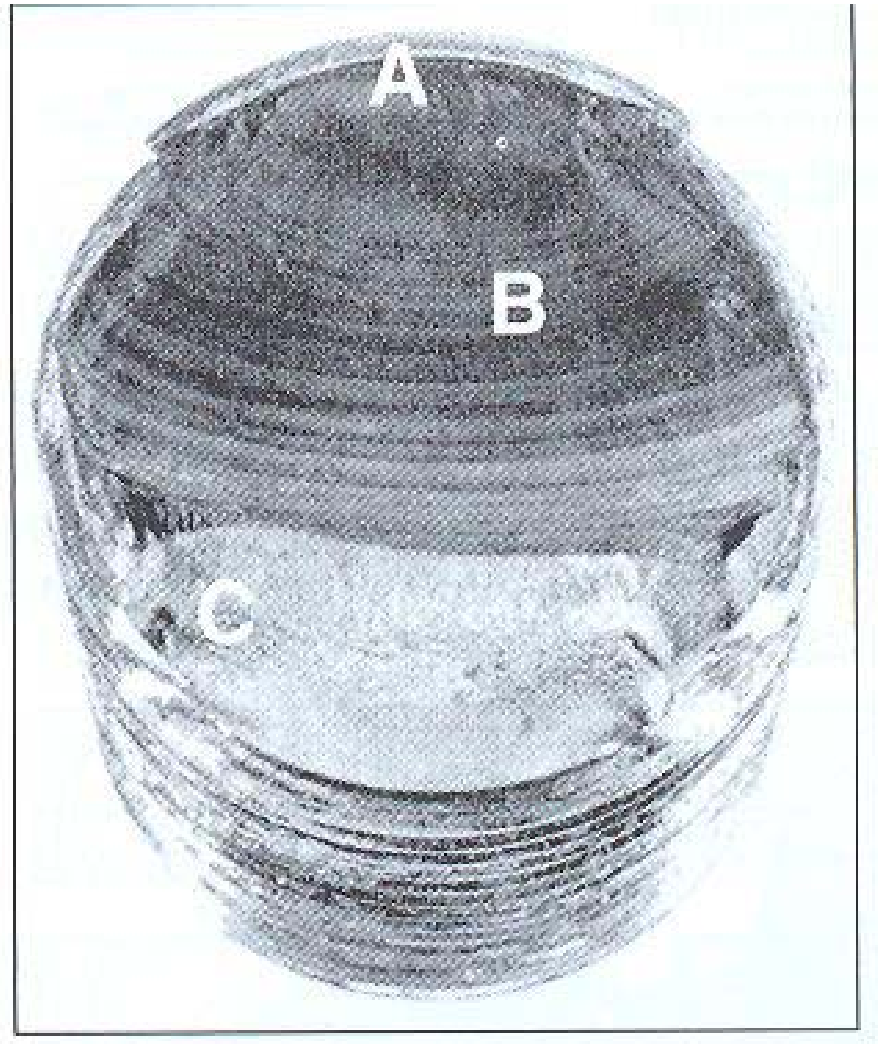
Alireza Safikhani

Fatigue …

Fatigue

Crack propagation

Fatigue failure of a bolt due to repeated unidirectional bending. The failure started at the thread root at *A*, propagated across most of the cross section shown by the beach marks at *B*, before final fast fracture at *C*. *(From ASM Handbook, Vol. 12: Fractography, ASM International, Materials Park, OH 44073-0002, fig 50, p. 120. Reprinted by permission of ASM International®, www.asminternational.org.)*



Crack propagation

Figure 7–4

Fatigue fracture surface of an AISI 8640 pin. Sharp corners of the mismatched grease holes provided stress concentrations that initiated two fatigue cracks indicated by the arrows. *(From ASM Handbook, Vol. 12: Fractography, ASM International, Materials Park, OH 44073-0002, fig 520, p. 331. Reprinted by permission of ASM International®, www.asminternational.org.)*

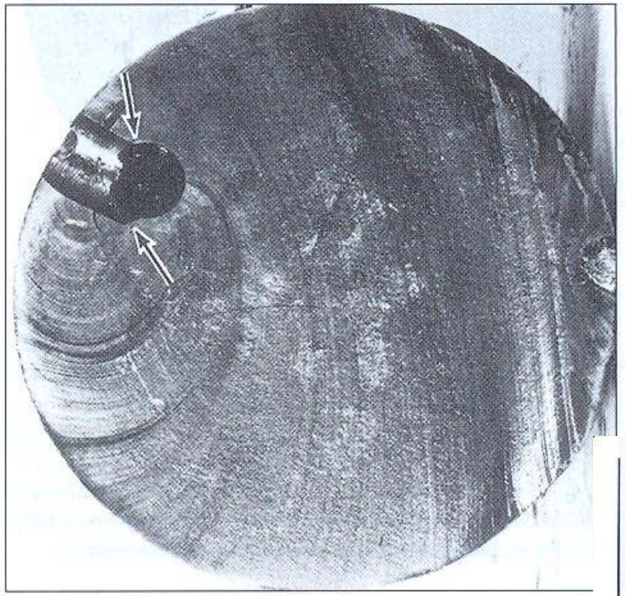
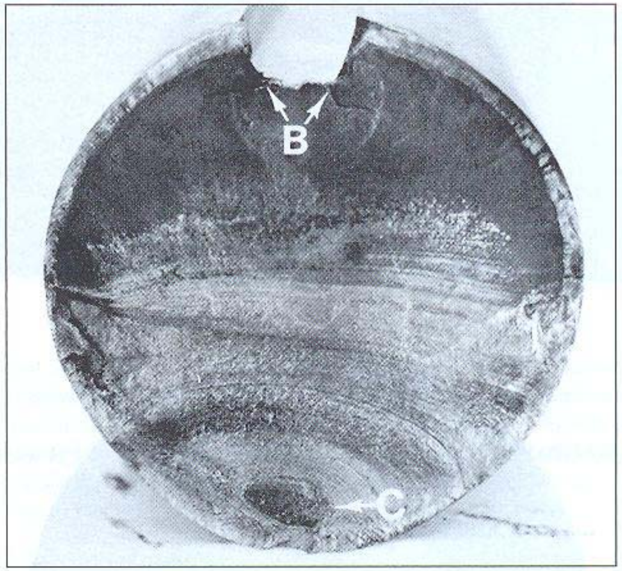
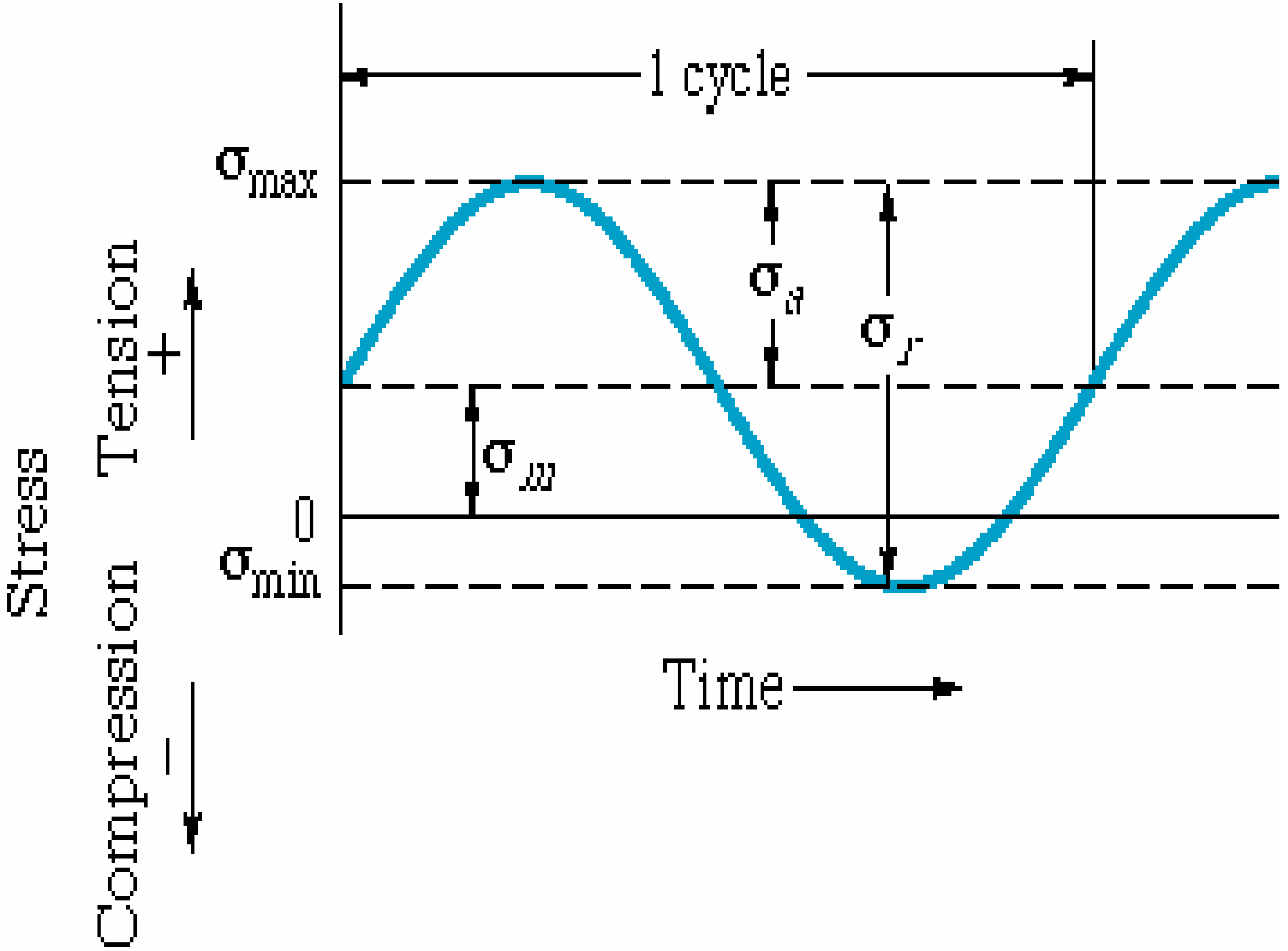


Figure 7–3

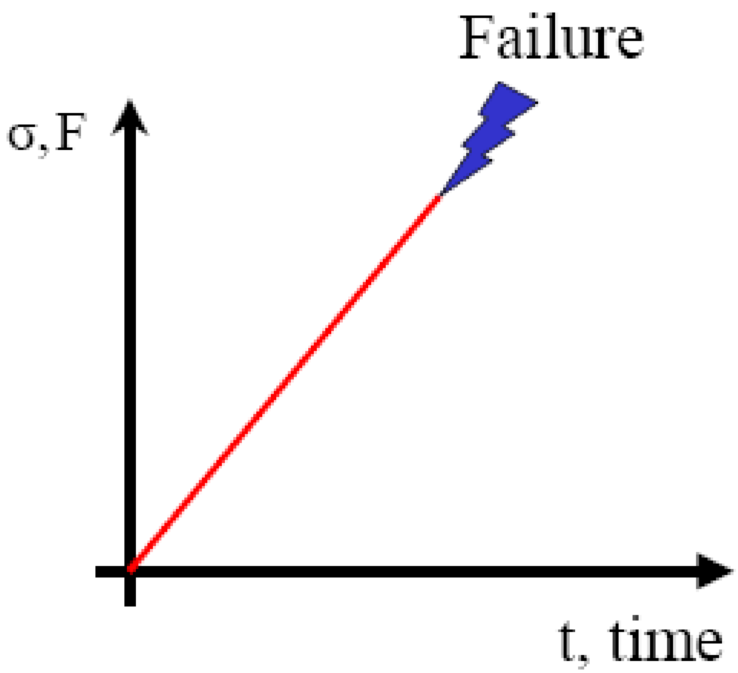
Fatigue fracture of an AISI 4320 drive shaft. The fatigue failure initiated at the end of the keyway at points *B* and progressed to final rupture at C. The final rupture zone is small, indicating that loads were low. *(From ASM Handbook, Vol. 11: Failure Analysis and Prevention, ASM International, Materials Park, OH 44073-0002, fig 18, p. 111. Reprinted by permission of ASM International®, www.asminternational.org.)*



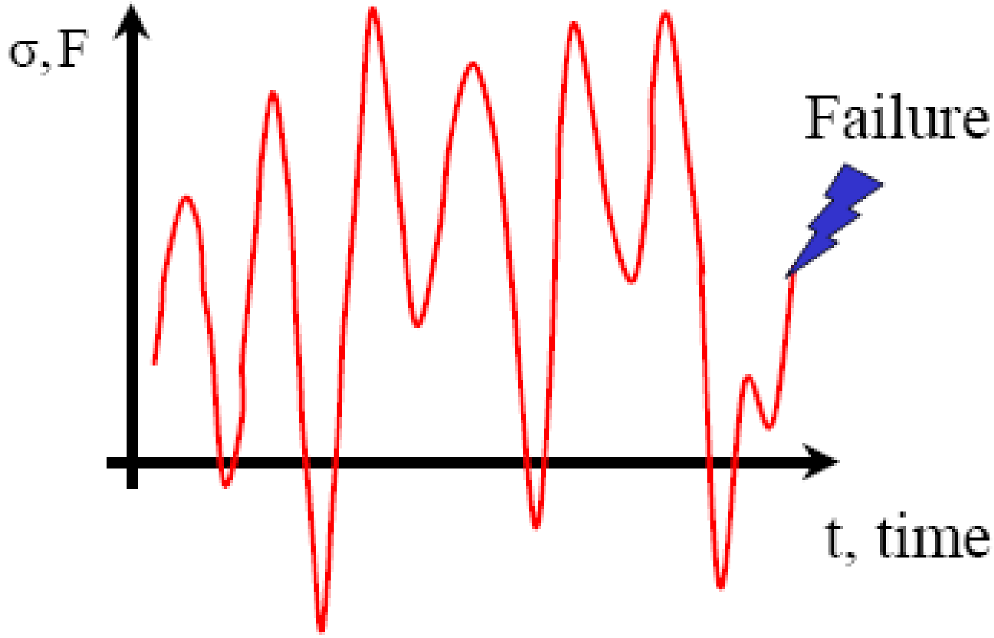
Cyclic Stress



Load Histories and Design Objectives



Monotonic, Static, or Steady



Dynamic, Cyclic, or Unsteady

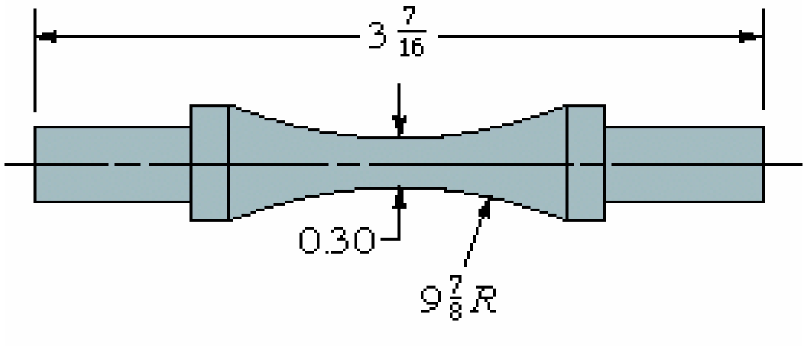
Design for Strength **Design for Life**

Fatigue Life Methods

- **The Stress-Life Method**

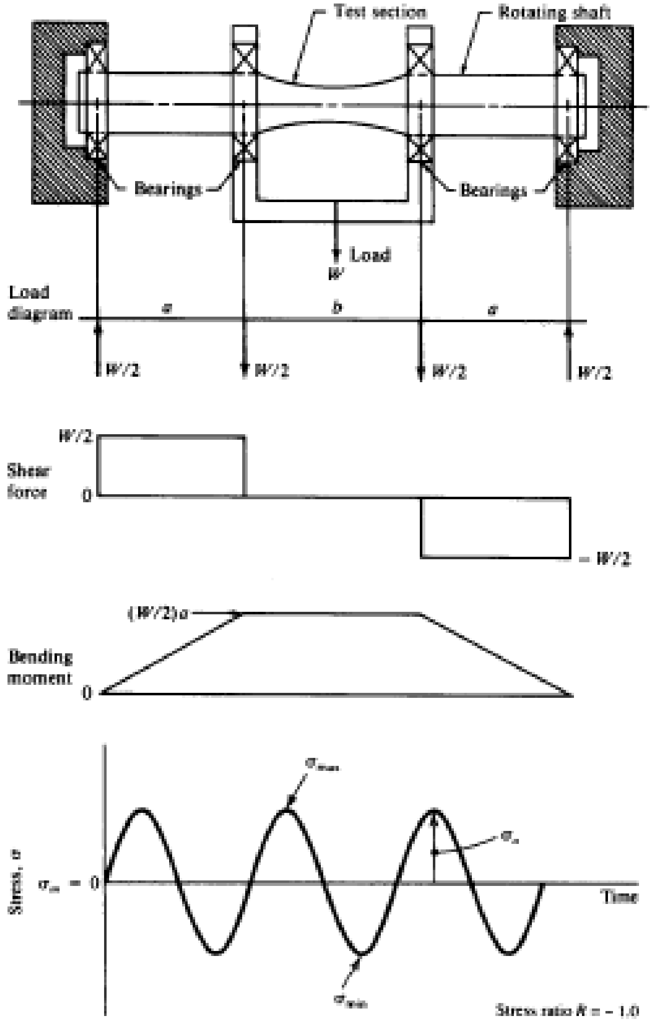
- **The Strain-Life Method**

R.R. Moore Specimen

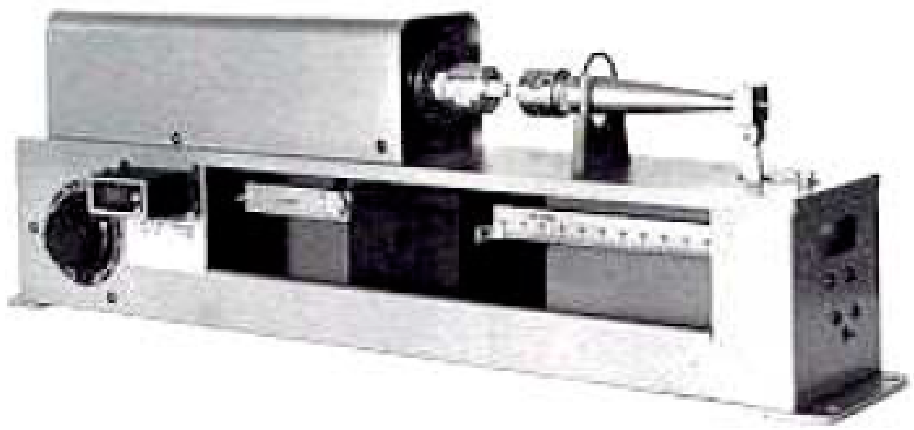


R.R. Moore machine fatigue test specimen.

Rotating Beam Fatigue Testing



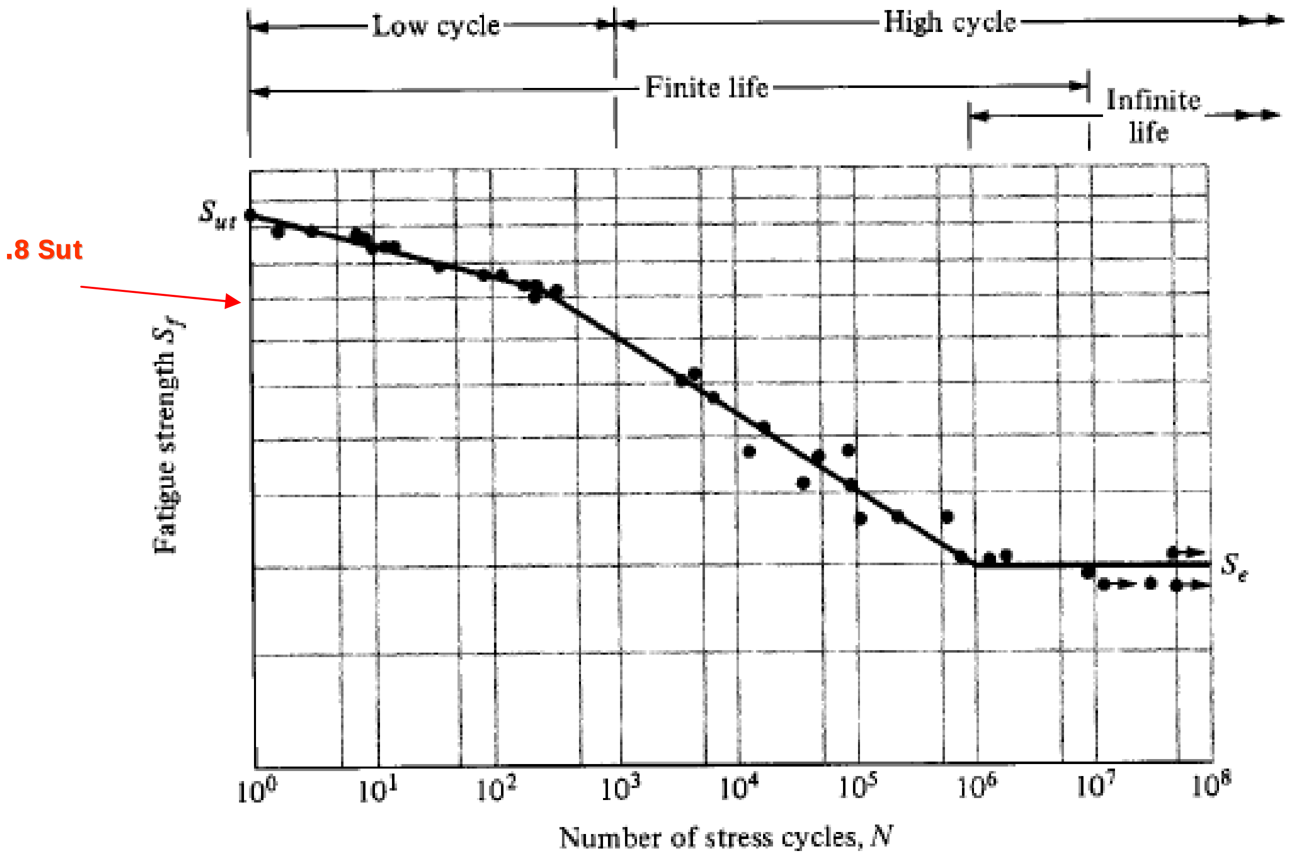
Mott, Fig. 5-2 & 5-3



Fatigue Dynamics, Inc. rotating beam test equipment.

www.fdinc.com

S-N Curve: The Stress-Life Method

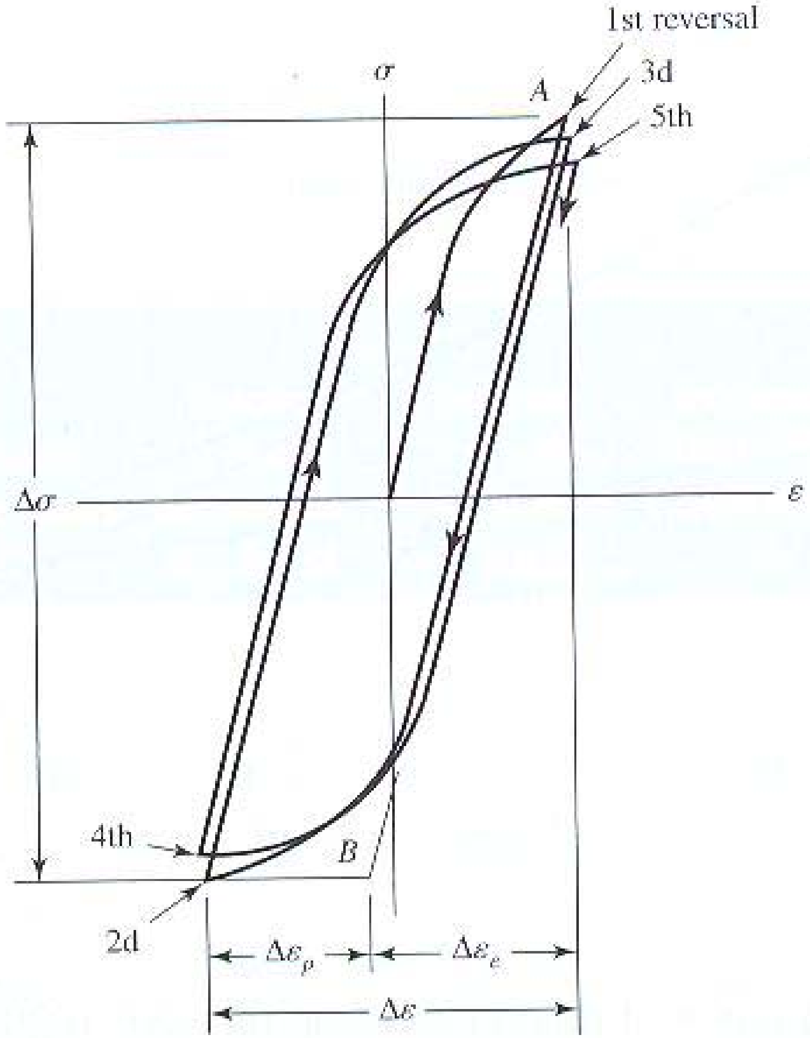


Completely reversed cyclic stress, UNS G41200 steel

Shigley, Fig. 7-6

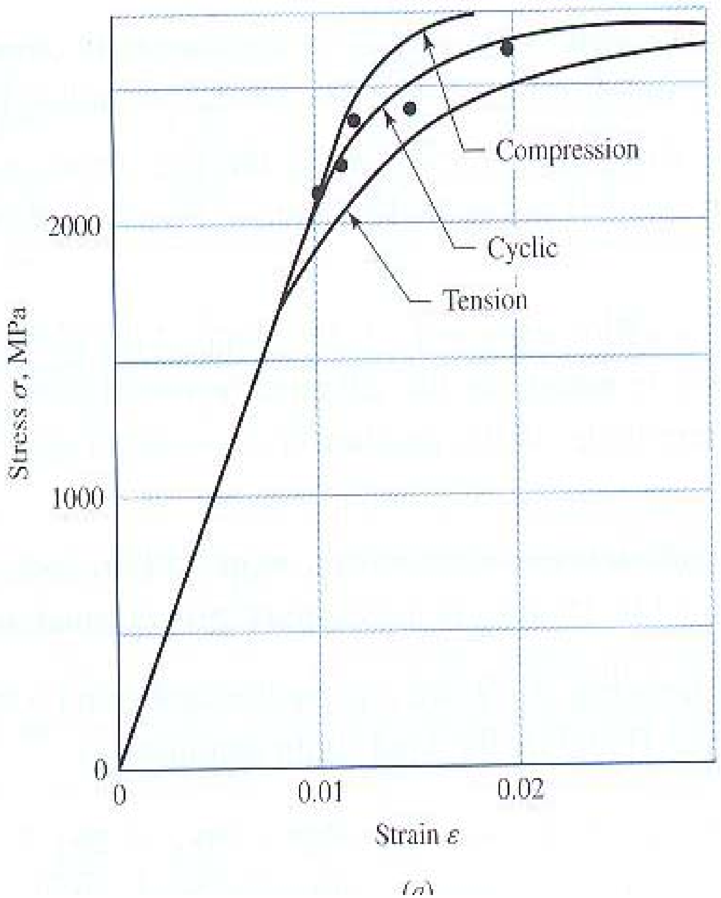
True stress- strain cycle loading

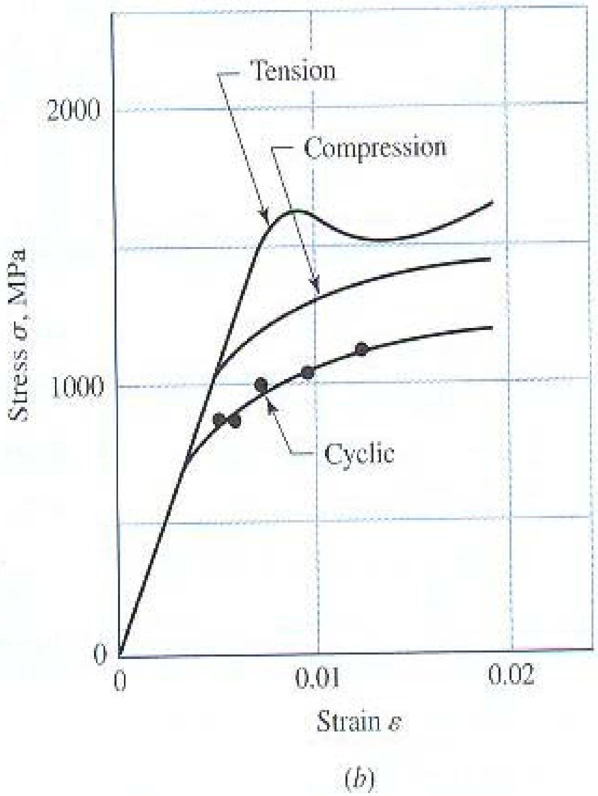
True stress–true strain hysteresis loops showing the first five stress reversals of a cyclic-softening material. The graph is slightly exaggerated for clarity. Note that the slope of the line *AB* is the modulus of elasticity *E*. The stress range is $\Delta\sigma$, $\Delta\varepsilon_p$ is the plastic-strain range, and $\Delta\varepsilon_e$ is the elastic strain range. The total-strain range is $\Delta\varepsilon = \Delta\varepsilon_p + \Delta\varepsilon_e$.



Cyclic loading

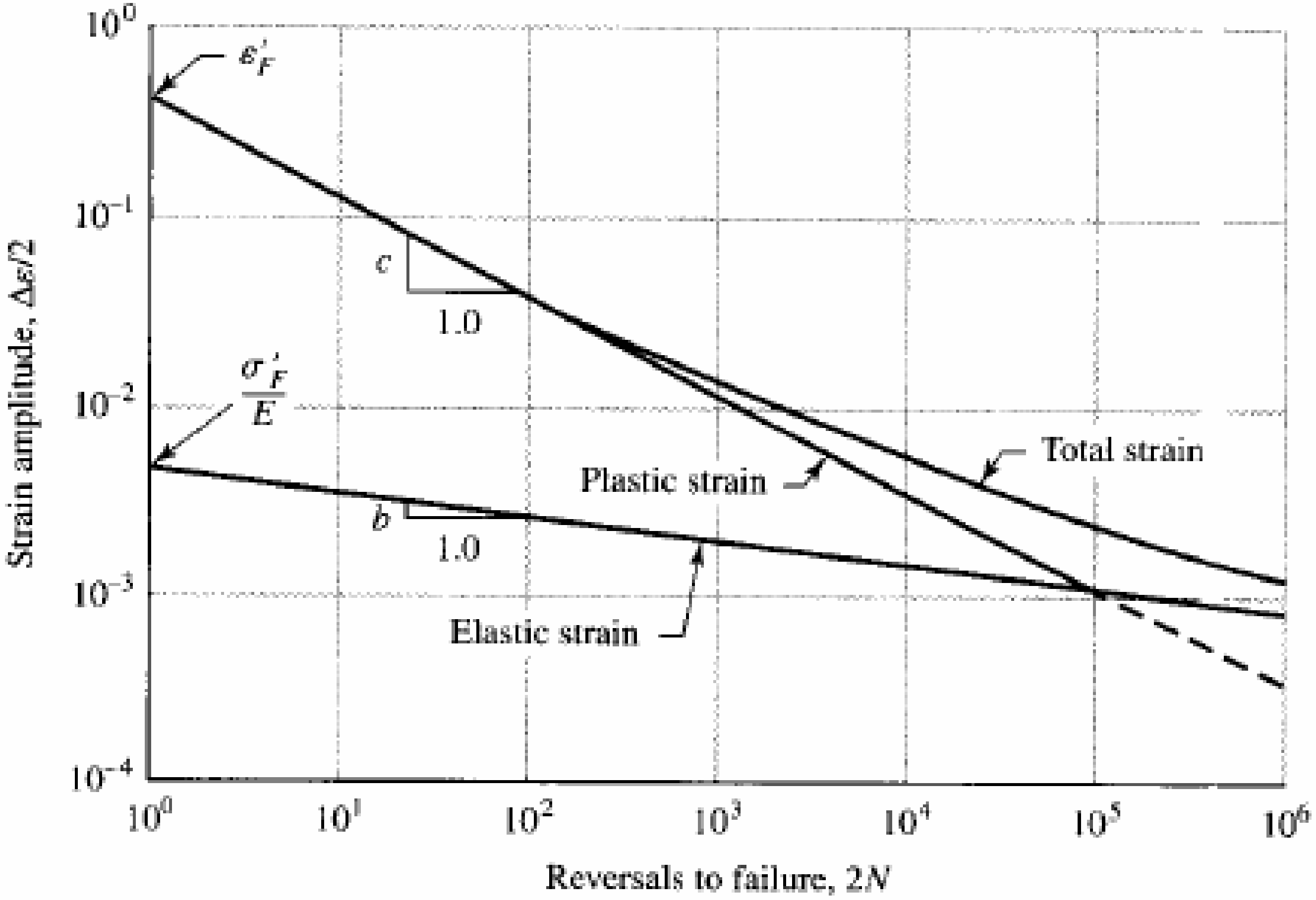
Monotonic and cyclic stress-strain results. (*a*) Ausformed H-11 steel, 660 Brinell; (*b*) SAE 4142 steel, 400 Brinell.





The Strain Life Method

A log-log plot showing how the fatigue life is related to the true-strain amplitude for hot-rolled SAE 1020 steel. *(Reprinted with permission from SAE J1099_200208 © 2002 SAE International.)*



Where…

- *Fatigue ductility coefficient* ε'_F is the true strain corresponding to fracture in one reversal (point A in Fig. 7–12). The plastic-strain line begins at this point in Fig. 7–14.
- *Fatigue strength coefficient* σ'_F is the true stress corresponding to fracture in one reversal (point A in Fig. 7–12). Note in Fig. 7–14 that the elastic-strain line begins at σ'_F / E.
- *Fatigue ductility exponent* c is the slope of the plastic-strain line in Fig. 7–14 and is the power to which the life $2N$ must be raised to be proportional to the true plastic-strain amplitude. If the number of stress reversals is $2N$, then N is the number of cycles.
- *Fatigue strength exponent* b is the slope of the elastic-strain line, and is the power to which the life $2N$ must be raised to be proportional to the true-stress amplitude.

The Strain Life Method

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_p}{2} + \frac{\Delta\varepsilon_e}{2}$$

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f' (2N)^c$$

$$\frac{\Delta\varepsilon_e}{2} = \sigma_f' (2N)^b$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E} (2N)^b + \varepsilon_f' (2N)^c$$

which is the Manson-Coffin relationship between fatigue life and total strain

Cyclic Properties of Some Metals

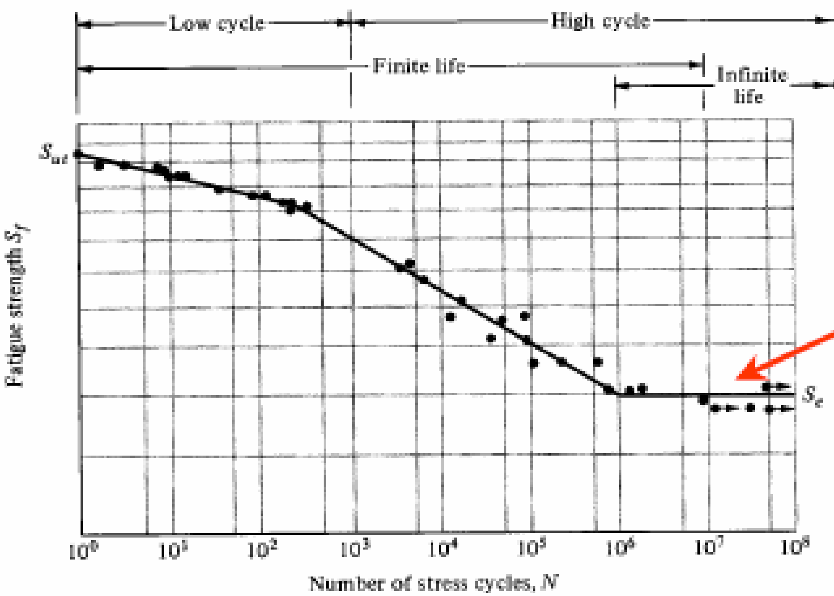
AISI Number	Processing	Brinell Hardness H_B	Cyclic Yield Strength S'_y, kpsi	Fatigue Strength Coefficient σ'_F, kpsi	Fatigue Ductility Coefficient ε'_F	Fatigue Strength Exponent b	Fatigue Ductility Exponent c	Fatigue Strain-Hardening Exponent m
1045	Q & T 80°F	705	. . .	310	. . .	−0.065	−1.0	0.10
1045	Q & T 360°F	595	250	395	0.07	−0.055	−0.60	0.13
1045	Q & T 500°F	500	185	330	0.25	−0.08	−0.68	0.12
1045	Q & T 600°F	450	140	260	0.35	−0.07	−0.69	0.12
1045	Q & T 720°F	390	110	230	0.45	−0.074	−0.68	0.14
4142	Q & T 80°F	670	300	375	. . .	−0.075	−1.0	0.05
4142	Q & T 400°F	560	250	385	0.07	−0.076	−0.76	0.11
4142	Q & T 600°F	475	195	315	0.09	−0.081	−0.66	0.14
4142	Q & T 700°F	450	155	290	0.40	−0.080	−0.73	0.12
4142	Q & T 840°F	380	120	265	0.45	−0.080	−0.75	0.14
4142*	Q & D 550°F	475	160	300	0.20	−0.082	−0.77	0.12
4142	Q & D 650°F	450	155	305	0.60	−0.090	−0.76	0.13
4142	Q & D 800°F	400	130	275	0.50	−0.090	−0.75	0.14

*Deformed 14 percent.

Fatigue Strength

The **Fatigue Strength**, **$S_f(N)$**, is the stress level that a material can endure for N cycles.

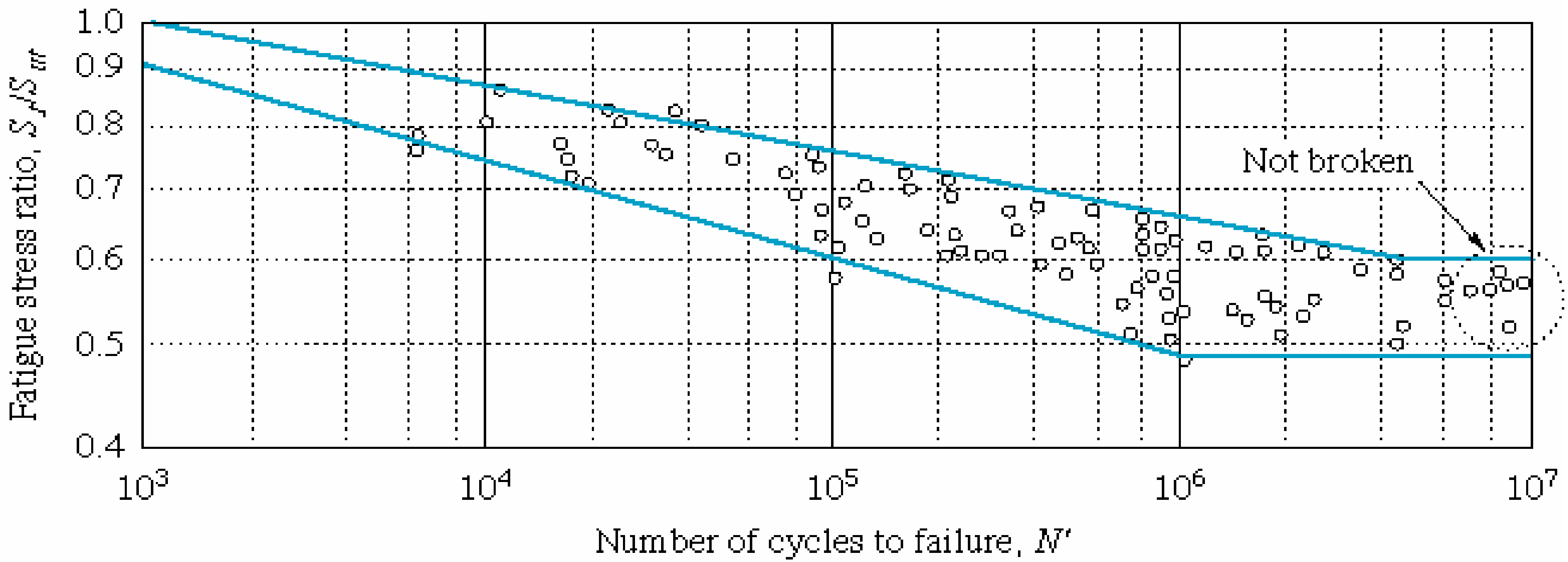
The stress level at which the material can withstand an infinite number of cycles is call the **Endurance Limit**.



The Endurance Limit is observed as a horizontal line on the S-N curve.

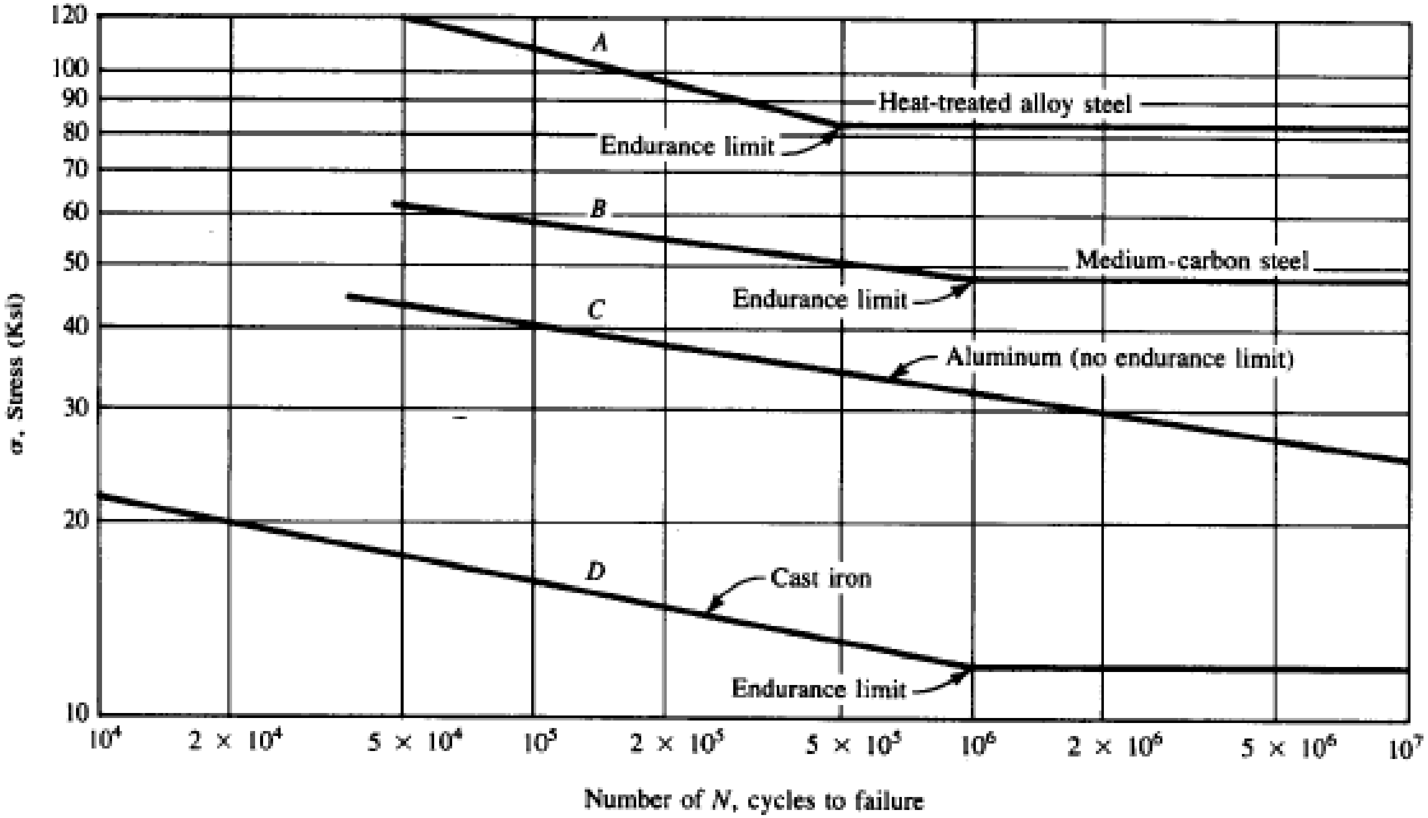
Shigley, Fig. 7-6

Fatigue Strength vs. Cycles to Failure



(a)

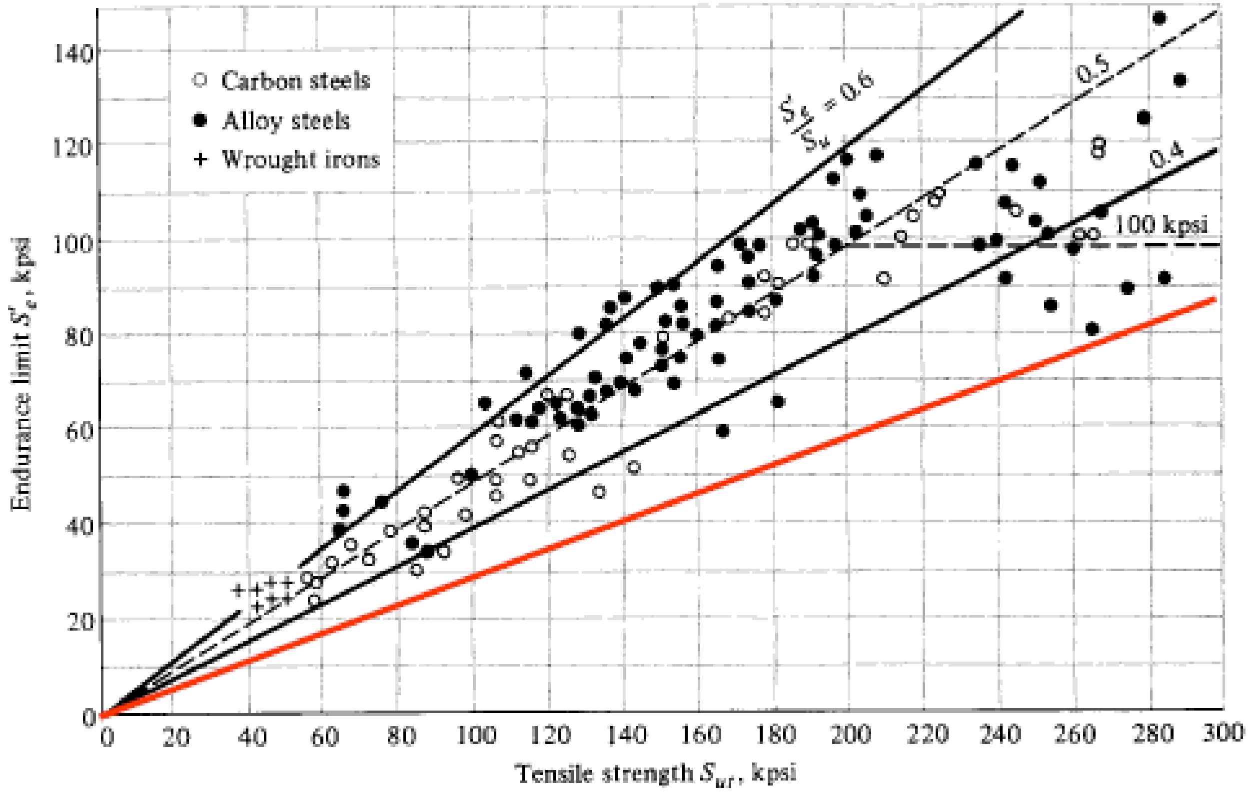
Representative S-N Curves



Note that non-ferrous materials often exhibit no endurance limit.

Mott, Fig. 5-7

Endurance Limit Vs Tensile Strength



Conservative Lower Bound for Ferrous Materials

$$S'_e = 0.3 S_{ut}$$

$$S'_e \equiv \text{Endurance Limit of Test Specimen}$$

$$S_{ut} \equiv \text{Tensile Strength of Test Specimen}$$

Shigley, Fig. 7-7

Approximate Endurance Limit for Various Materials

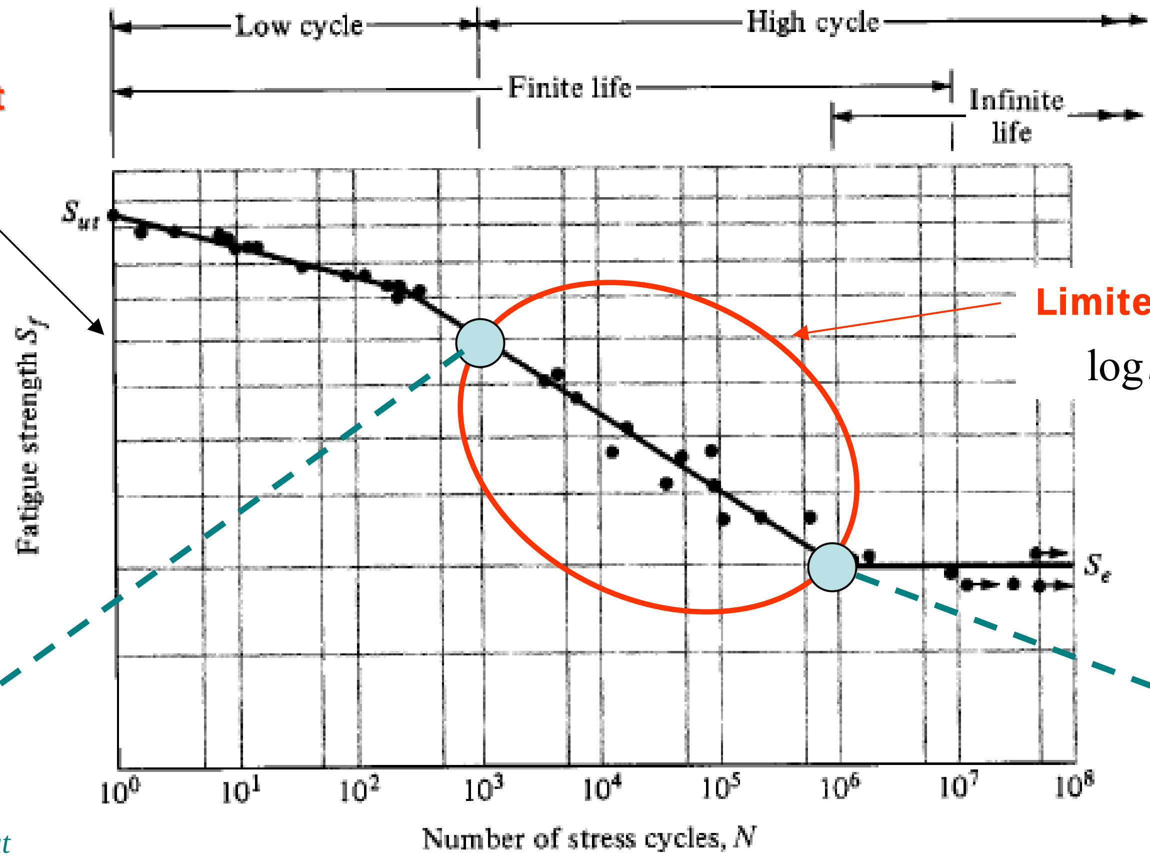
Material	Number of Cycles	Relation
Magnesium alloys	10^8	$S'_e = 0.35 S_u$
Copper alloys	10^8	$0.25 S_u < S'_e < 0.5\, S_u$
Nickel alloys	10^8	$0.35\, S_u < S'_e < 0.65\, S_u$
Titanium	10^7	$0.45\, S_u < S'_e < 0.65\, S_u$
Aluminum alloys	5×10^8	$S'_e = 0.45\, S_u$ ($S_u < 48$ksi) $S'_e = 19$ ksi ($S_u \geq 48$ksi)

For steel

$$S'_e = \begin{cases} 0.504 S_{ut} \text{ kpsi or MPa} & S_{ut} \leq 212 \text{ kpsi (1460 MPa)} \\ 107 \text{ kpsi} & S_{ut} > 212 \text{ kpsi} \\ 740 \text{ MPa} & S_{ut} > 1460 \text{ MPa} \end{cases}$$

Fatigue Life: Limited Life

.8 Sut Or f.Sut



Limited Life Region

$$\log S_f = b \log N + C$$

$$N = 10^3$$
$$S_f = 0.8 S_{ut}$$

$$N = 10^6$$
$$S_f = S_e'$$

Completely reversed cyclic stress, UNS G41200 steel

Shigley, Fig. 7-6

Fatigue Strength

$$\log S_f = b \log N + C \,,$$

$$N = 10^3 \qquad S_f = 0.8 S_{ut}$$

$$N = 10^6 \qquad S_f = S'_e$$

$$\Rightarrow \begin{cases} b = -\dfrac{1}{3}\log \dfrac{0.8S_{ut}}{S'_e} \\ c = \log \dfrac{(0.8S_{ut})^2}{S'_e} \end{cases}$$

$$\left.\begin{array}{l} S_f \longrightarrow N = 10^{-\frac{c}{b}} S_f^{\frac{1}{b}} \\ N \longrightarrow S_f = 10^c N^b \end{array}\right\} \quad 10^3 < N < 10^6$$

Example

- **Please Write!!!!!**

Endurance Limit Multiplying Factors (Marin Factors)

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

There are several factors that are known to result in differences between the endurance limits in test specimens and those found in machine elements.

where k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[15]

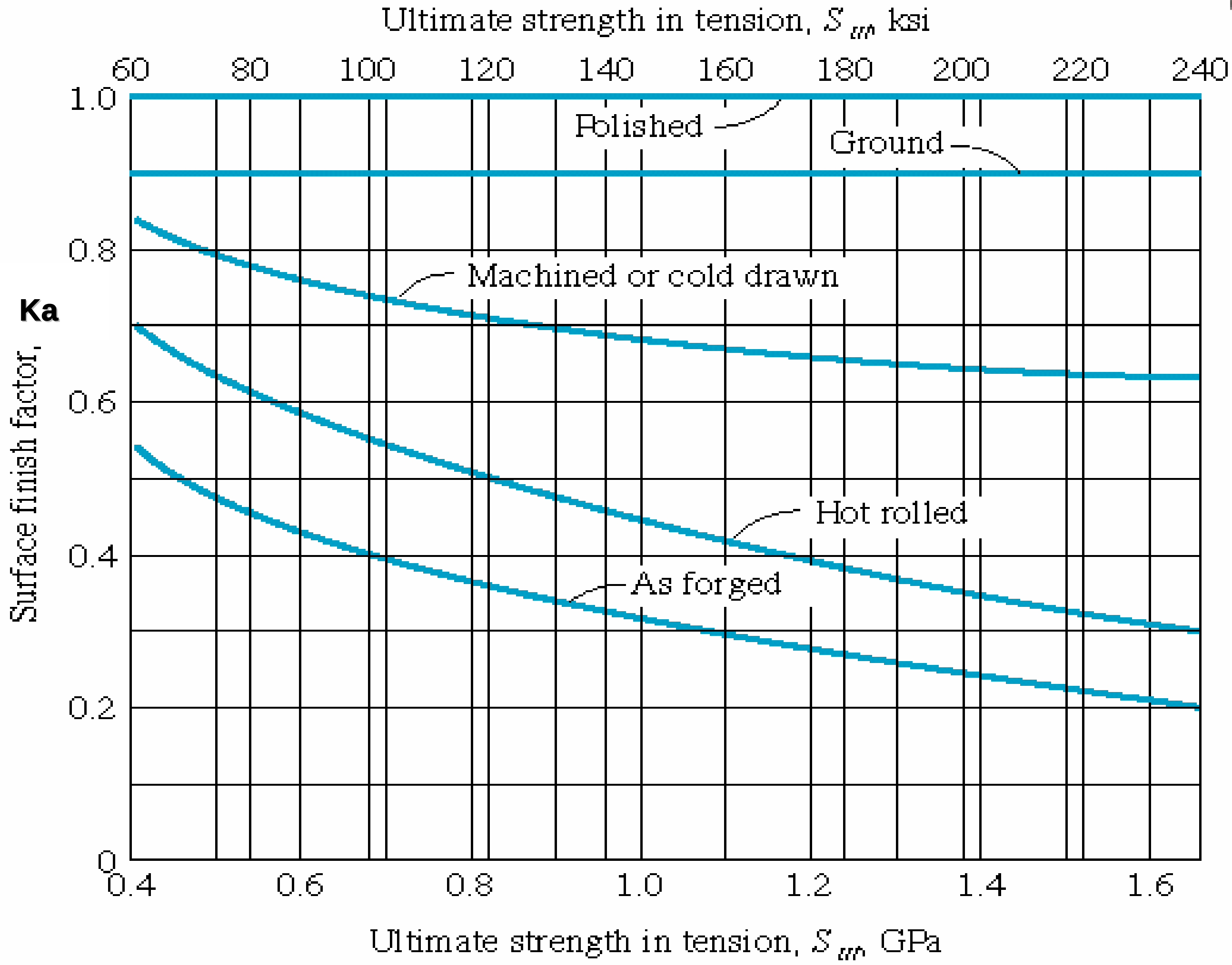
k_f = miscellaneous-effects modification facto.

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

When endurance tests of parts are not available, estimations are made by applying Marin factors to the endurance limit.

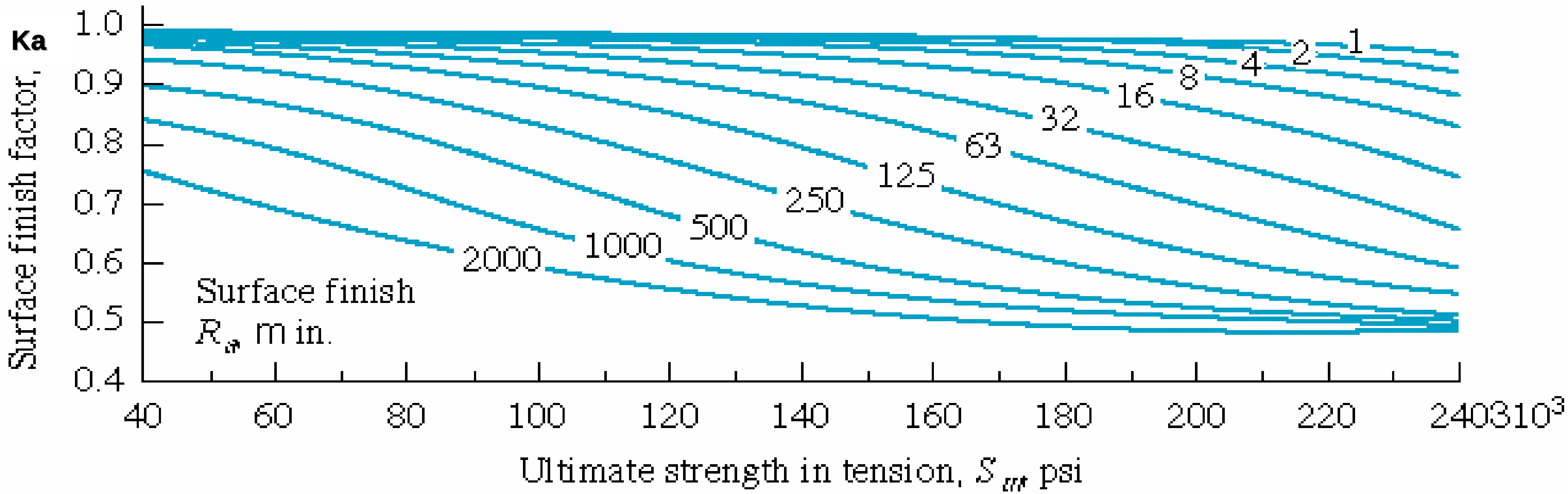
Surface Finish Factors: Ka



(a)

Surface finish factors for steel (a) Function of ultimate strength in tension for different machine processes.

Surface Finish Factors (cont.): Ka



(b)

Surface finish factors for steel (b) Function of ultimate strength and surface roughness as measured with a stylus profilometer. [From Johnson (1967).]

Surface Finish Factor: Ka

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged **& Welding**	39.9	272.	−0.995

$$K_a = a(S_{ut})^b$$

EXAMPLE 7–3 A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

Solution From Table 7–4, $a = 4.51$ and $b = -0.265$. Then, from Eq. (7–18)

$$k_a = 4.51(520)^{-0.265} = 0.860$$

Size Factor: Kb

- **For torsion and Bending:**

$$K_b = \begin{cases} 1.24d^{-0.107} & 2.79 \le d \le 51\ mm \\ 1.51d^{-0.157} & 51 < d \le 254 \end{cases}$$

- **For axial loading there is no size effect, so**

$$K_b = 1$$

- **One of the problems that arise in using this equ. Is what to do when a round bar in bending is not rotating or when a noncircular cross section is used ?**

Kb for noncircular or nonrotating circular

what is the size factor for a bar 6 mm thick and 40 mm wide? The approach to be used here employs an *effective dimension* d_e obtained by equating the volume of material stressed at and above 95 percent of the maximum stress to the same volume in the rotating-beam specimen.[18] It turns out that when these two volumes are equated, the lengths cancel, and so we need only consider the areas. For a rotating round section, the 95 percent stress area is the area in a ring having an outside diameter d and an inside diameter of $0.95d$. So, designating the 95 percent stress area $A_{0.95\sigma}$, we have

$$A_{0.95\sigma} = \frac{\pi}{4}[d^2 - (0.95d)^2] = 0.0766d^2 \tag{7-21}$$

This equation is also valid for a rotating hollow round.

Kb for noncircular or nonrotating circular

This equation is also valid for a rotating hollow round. For nonrotating solid or hollow rounds, the 95 percent stress area is twice the area outside of two parallel chords having a spacing of $0.95d$, where d is the diameter. Using an exact computation, this is

$$A_{0.95\sigma} = 0.01046d^2 \tag{7–22}$$

with d_e in Eq. (7–21), setting Eqs. (7–21) and (7–22) equal to each other enables us to solve for the effective diameter. This gives

$$d_e = 0.370d \tag{7–23}$$

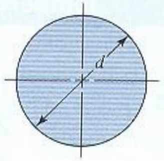
as the effective size of a round corresponding to a nonrotating solid or hollow round.

A rectangular section of dimensions $h \times b$ has $A_{0.95\sigma} = 0.05hb$. Using the same approach as before,

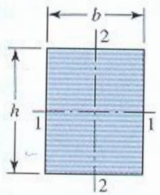
$$d_e = 0.808(hb)^{1/2} \tag{7–24}$$

Kb for noncircular or nonrotating circular

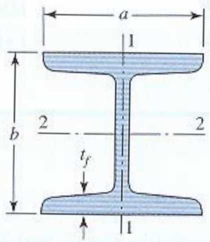
Table 7–5 provides $A_{0.95\sigma}$ areas of common structural shapes undergoing non-rotating bending.



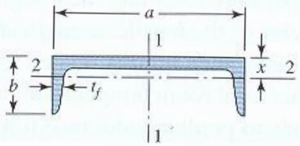
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & & \text{axis 1-1} \\ 0.05ba & t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

Example

EXAMPLE 7–4 A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor k_b if the shaft is used in

(*a*) A rotating mode.

(*b*) A nonrotating mode.

Solution (*a*) From Eq. (7–19),

Answer

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{32}{7.62}\right)^{-0.107} = 0.858$$

(*b*) From Table 7–5,

$$d_e = 0.37d = 0.37(32) = 11.84 \text{ mm}$$

From Eq. (7–19),

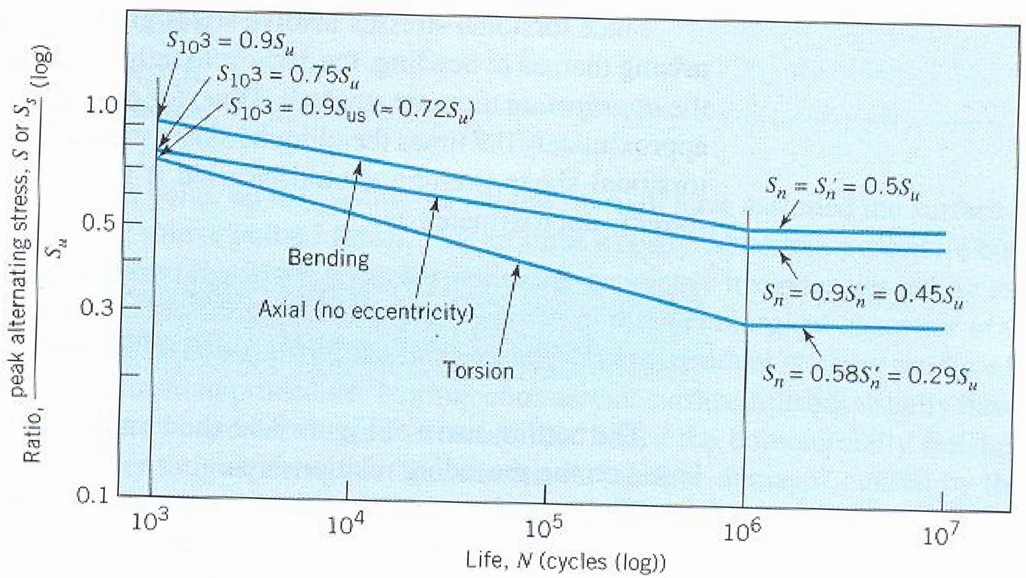
Answer

$$k_b = \left(\frac{11.84}{7.62}\right)^{-0.107} = 0.954$$

Load factor: Kc

When fatigue tests are carried out with rotating bending, axial (push-pull), and torsional loading, the endurance limits differ with S_{ut}.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{19} \end{cases}$$



URE 8.11
ıeralized S–N curves for
ished 0.3-in.-diameter
ɜl specimens (based on
culated elastic stresses,
oring possible yielding).

Temperature Factor: Kd

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \le \hat{\sigma} \le 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 3–8.

$$K_d = \begin{cases} 1 & T \le 350^\circ \\ 0.5 & 350 < T \le 500 \end{cases}$$

Temperature Factor: Kd

Table 7–6 has been obtained from Fig. 3–8 by using only the tensile-strength data. Note that the table represents 145 tests of 21 different carbon and alloy steels. A fourth-order polynomial curve fit to the data underlying Fig. 3–8 gives

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \qquad (7\text{–}26)$$

where $70 \leq T_F \leq 1000°$F.

Two types of problems arise when temperature is a consideration. If the rotating-beam endurance limit is known at room temperature, then use

$$k_d = \frac{S_T}{S_{RT}} \qquad (7\text{–}27)$$

from Table 7–6 or Eq. (7–26) and proceed as usual. If the rotating-beam endurance limit is not given, then compute it using Eq. (7–8) and the temperature-corrected tensile strength obtained by using the factor from Table 7–6. Then use $k_d = 1$.

Reliability factor: Ke

Reliability, %	Reliability Factor k_e
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620

Miscellaneous effects: Kf

- **Residual stress**
- **Corrosion**
- **Electrolytic Plating**

  Metallic coatings, such as chromium plating, nickel plating, or cadmium plating, reduce the endurance limit by as much as 50 percent. In some cases the reduction by coatings has been so severe that it has been necessary to eliminate the plating process. Zinc plating does not affect the fatigue strength. Anodic oxidation of light alloys reduces bending endurance limits by as much as 39 percent but has no effect on the torsional endurance limit.

- **Metal Spraying**

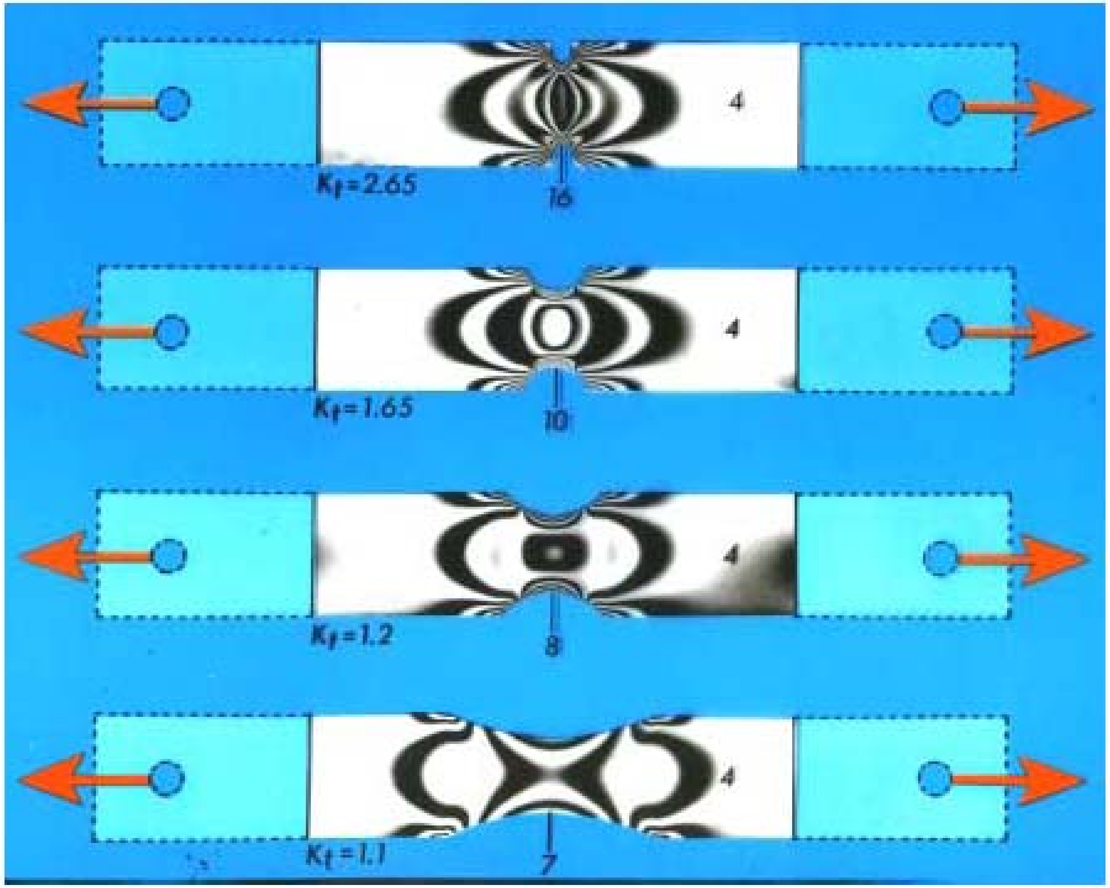
  Metal spraying results in surface imperfections that can initiate cracks. Limited tests show reductions of 14 percent in the fatigue strength.

Miscellaneous effects: Kf

- **Frettage Corrosion**

The phenomenon of frettage corrosion is the result of microscopic motions of tightly fitting parts or structures. Bolted joints, bearing-race fits, wheel hubs, and any set of tightly fitted parts are examples. The process involves surface discoloration, pitting, and eventual fatigue. The frettage factor k_f depends upon the material of the mating pairs and ranges from 0.24 to 0.90.

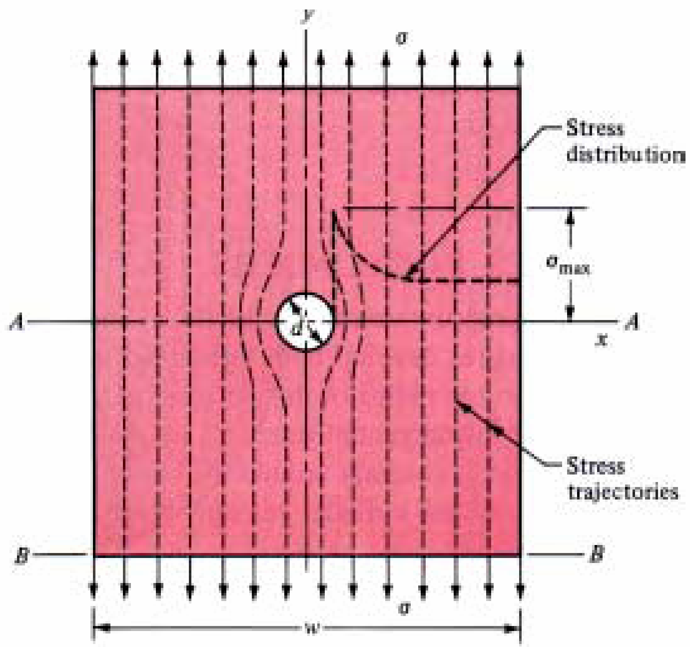
Stress Concentration and Notch Sensitivity



The discontinuity geometry has a significant effect on the stress distribution around it.

Vishay Lecture-Aid Series, LA-101

Geometric Stress Concentration Factors



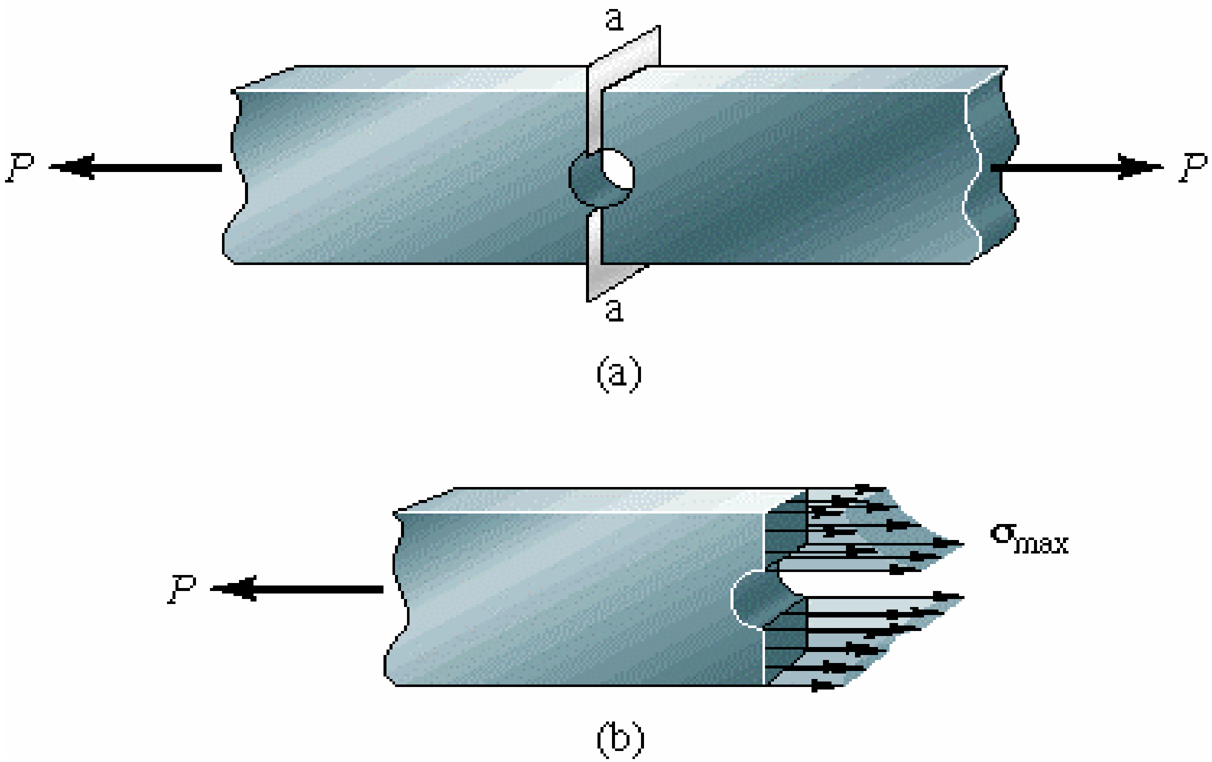
$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

$$\sigma_{nom} = \frac{F}{A_0}$$

$$A_0 = (w - d)t$$

Geometric stress concentration factors can be used to estimate the stress amplification in the vicinity of a geometric discontinuity.

Axial Load on Plate with Hole



Rectangular plate with hole subjected to axial load. (a) Plate with cross-sectional plane. (b) Half of plate with stress distribution.

Stress Concentrations for Bar with Fillet

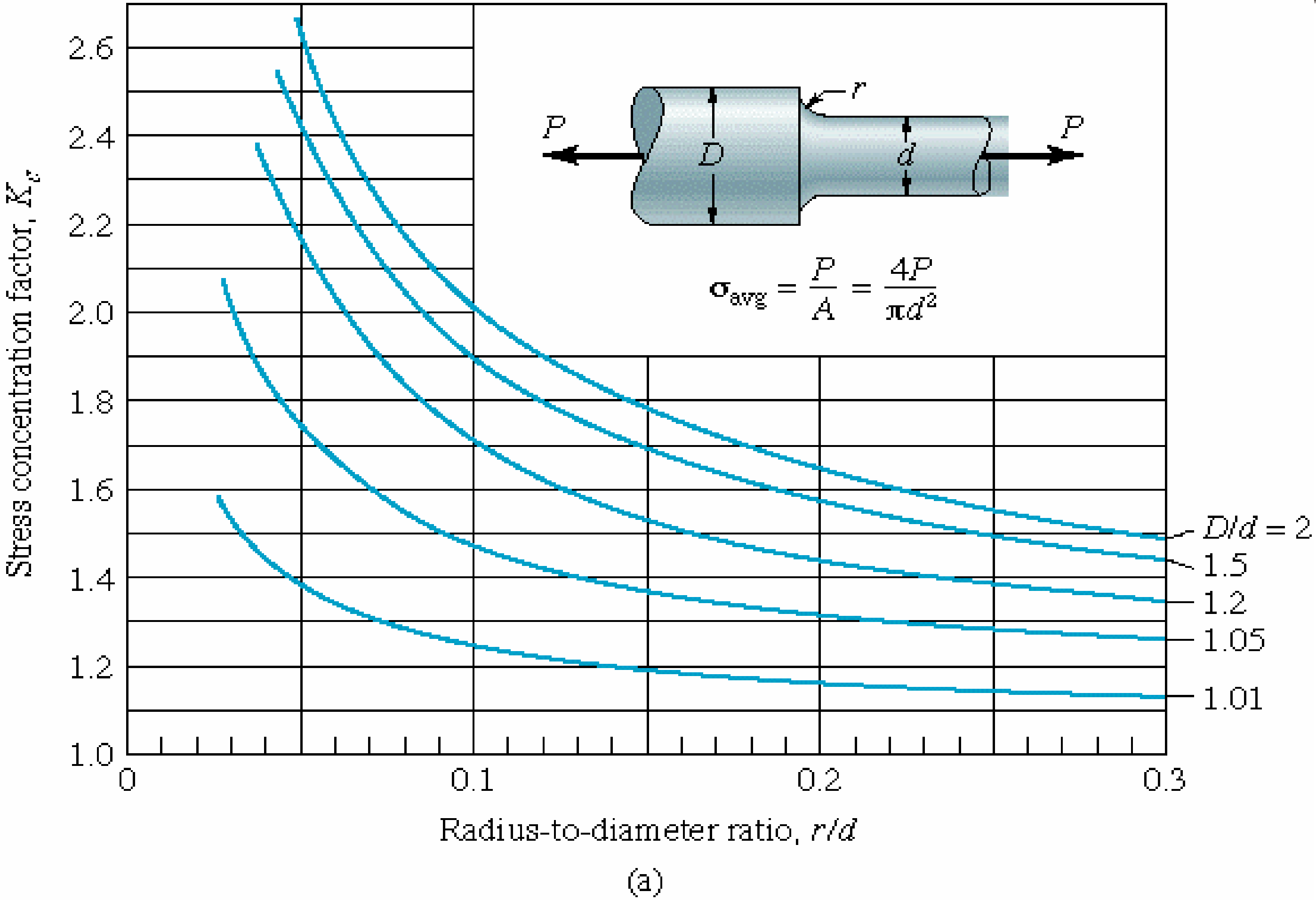


Figure 6.5 Stress concentration factor for round bar with fillet. (a) Axial load. [Adapted from Collins (1981).]

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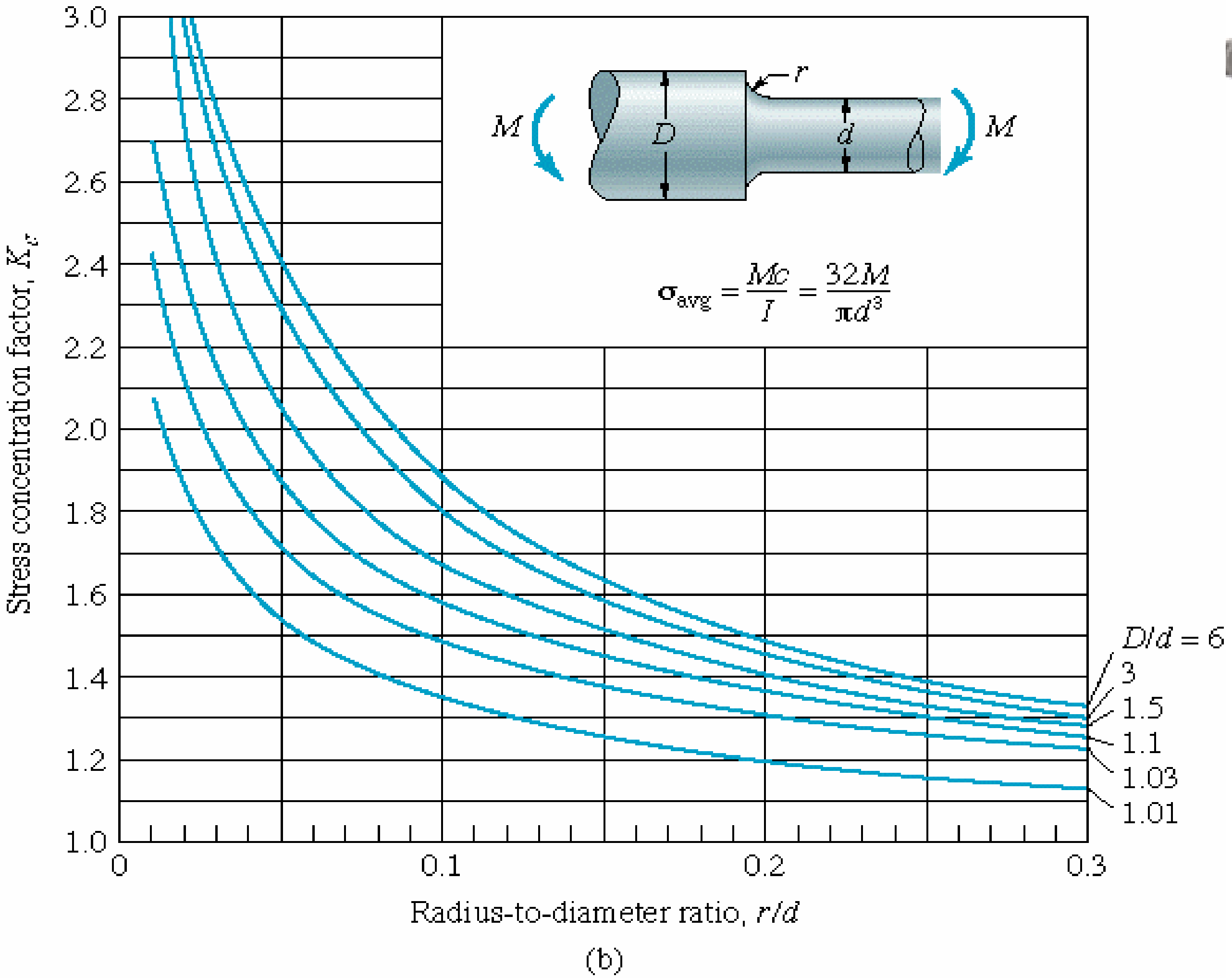
Stress Concentrations for Bar with Fillet (cont.)



Stress concentration factor for round bar with fillet. (b) Bending. [Adapted from Collins (1981).]

Stress Contours in Bar

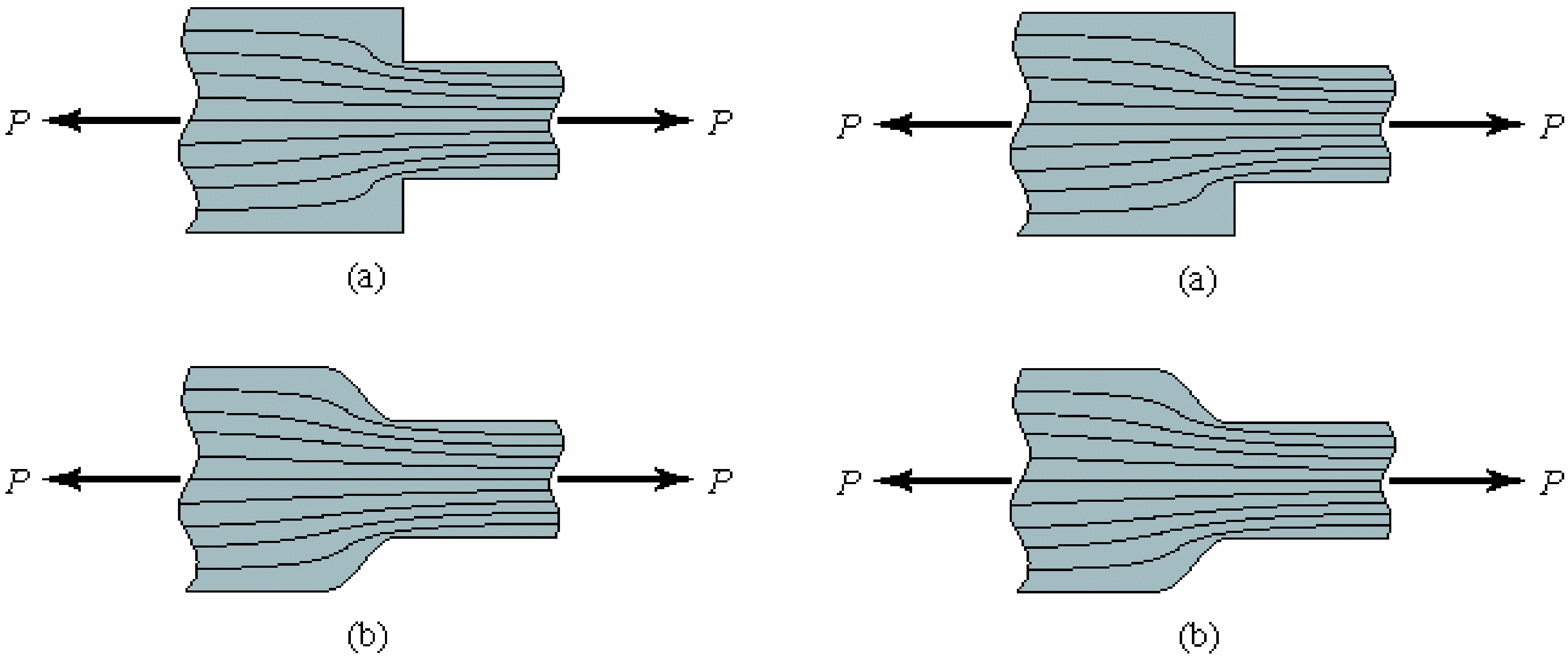


Figure 6.7 Bar with fillet axially loaded showing stress contours through a flat plate for (a) square corners, (b) rounded corners (c) small groove, and (d) small holes.

Geometric Stress Concentration Factors (Summary)

K_t is used to relate the maximum stress at the discontinuity to the nominal stress.

K_t is used for normal stresses

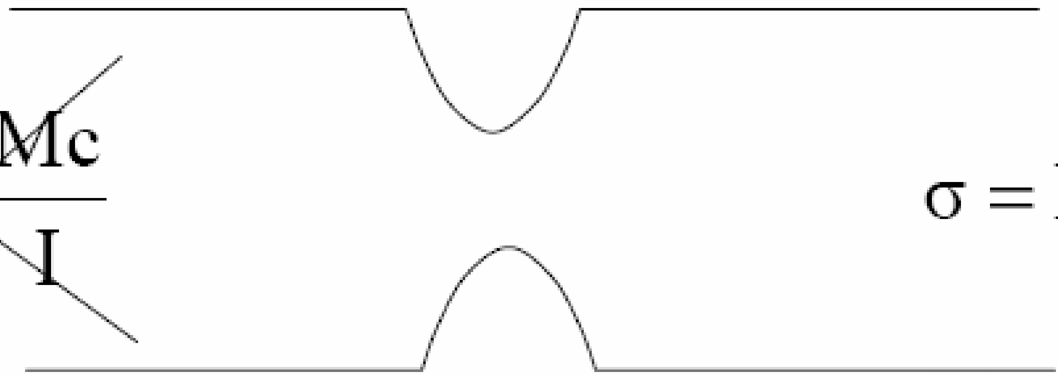
K_{ts} is used for shear stresses

K_t is based on the geometry of the discontinuity

σ_{nom} is usually computed using the minimum cross section

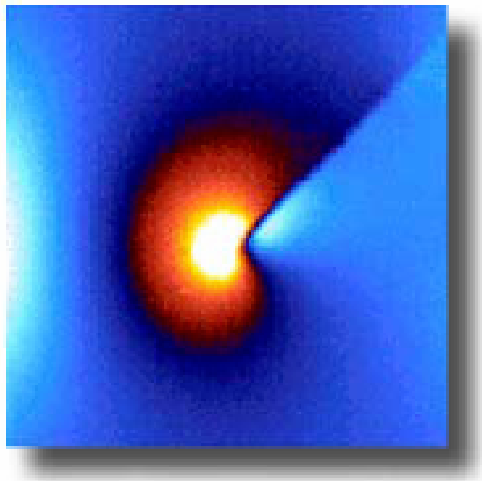
Un-notched and Notched Fatigue Specimens

$$\cancel{\sigma = K_t \frac{Mc}{I}}$$



$$\sigma = K_f \frac{Mc}{I}$$

Comparisons of fatigue test results for notched and un-notched specimens revealed that a reduced K_t was warranted for calculating the fatigue life for many materials.



www.stressphotonics.com

Fatigue Stress Concentration Factors

$$K_f = \frac{\text{Maximum stress in notched specimen}}{\text{Stress in notch - free specimen}}$$

or

$$K_f = \frac{\text{Endurance limit of a notched specimen.}}{\text{Endurance limit of a notch - free specimen.}}$$

Notch Sensitivity Factor

The notch sensitivity of a material is a measure of how sensitive a material is to notches or geometric discontinuities.

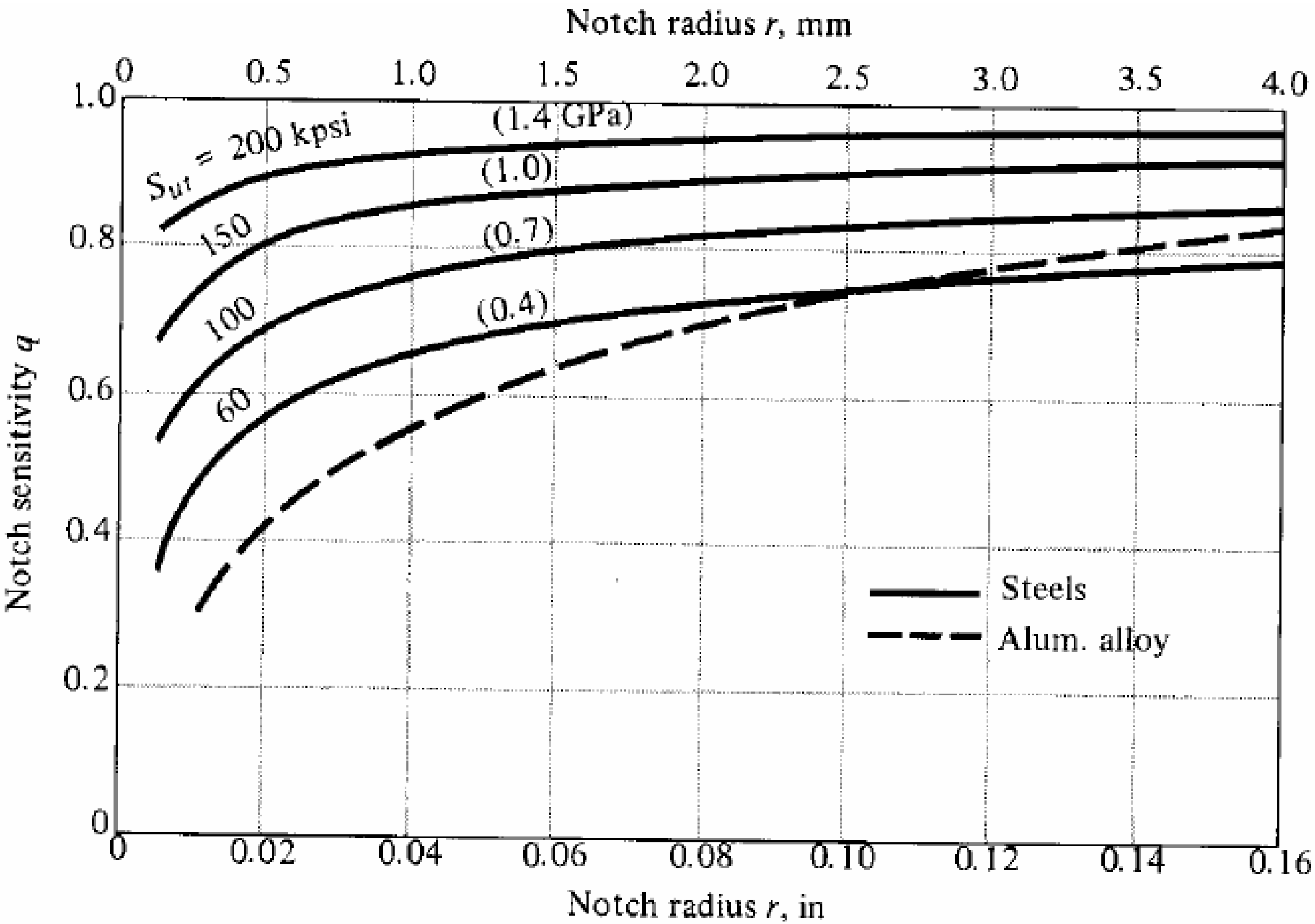
Fatigue stress Concentration Factor

$$q = \frac{K_f - 1}{K_t - 1} \qquad 0 \le q \le 1$$

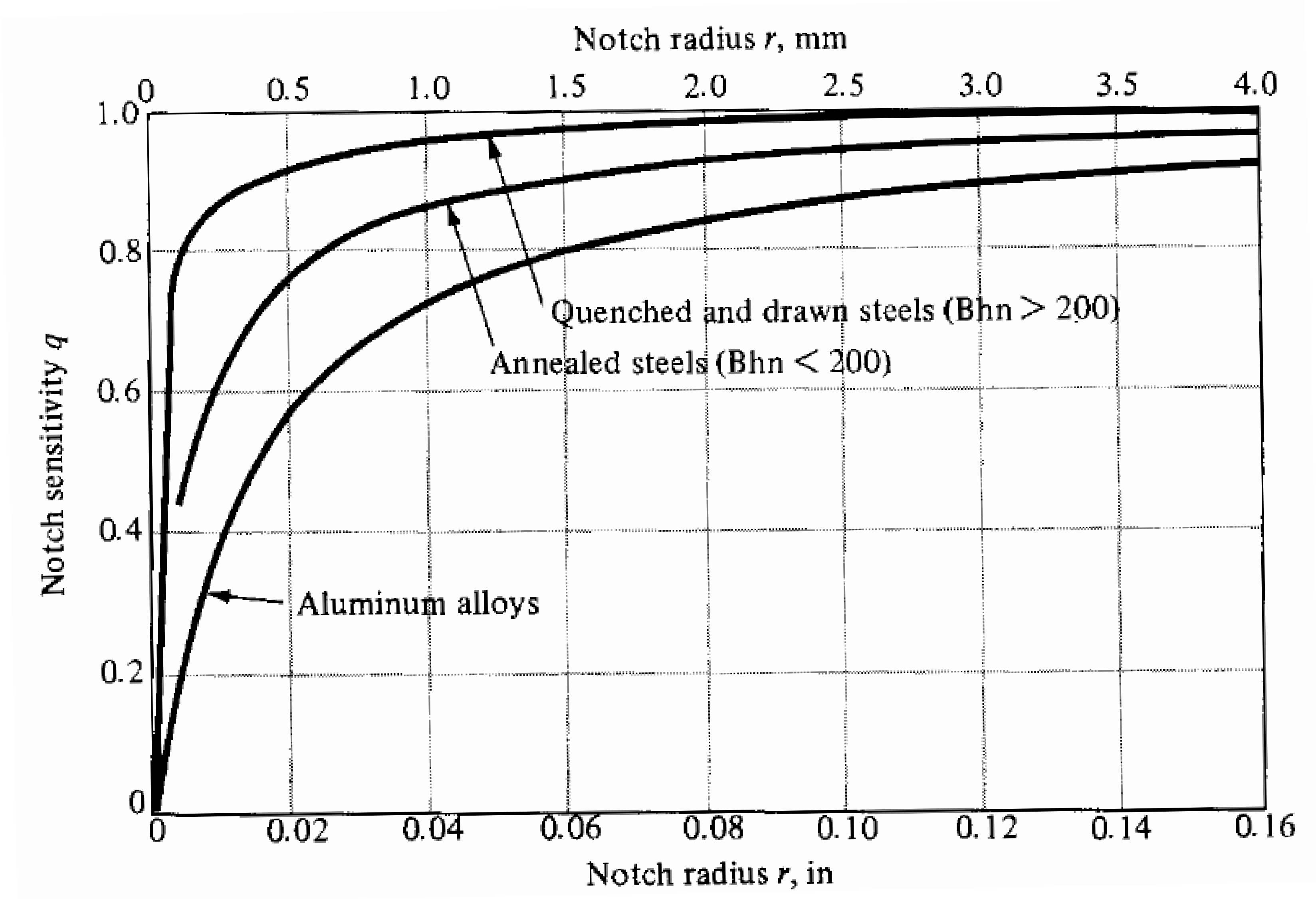
Static stress Concentration Factor

$$K_f = 1 + q\left(K_t - 1\right) \qquad 1 \le K_f \le K_t$$

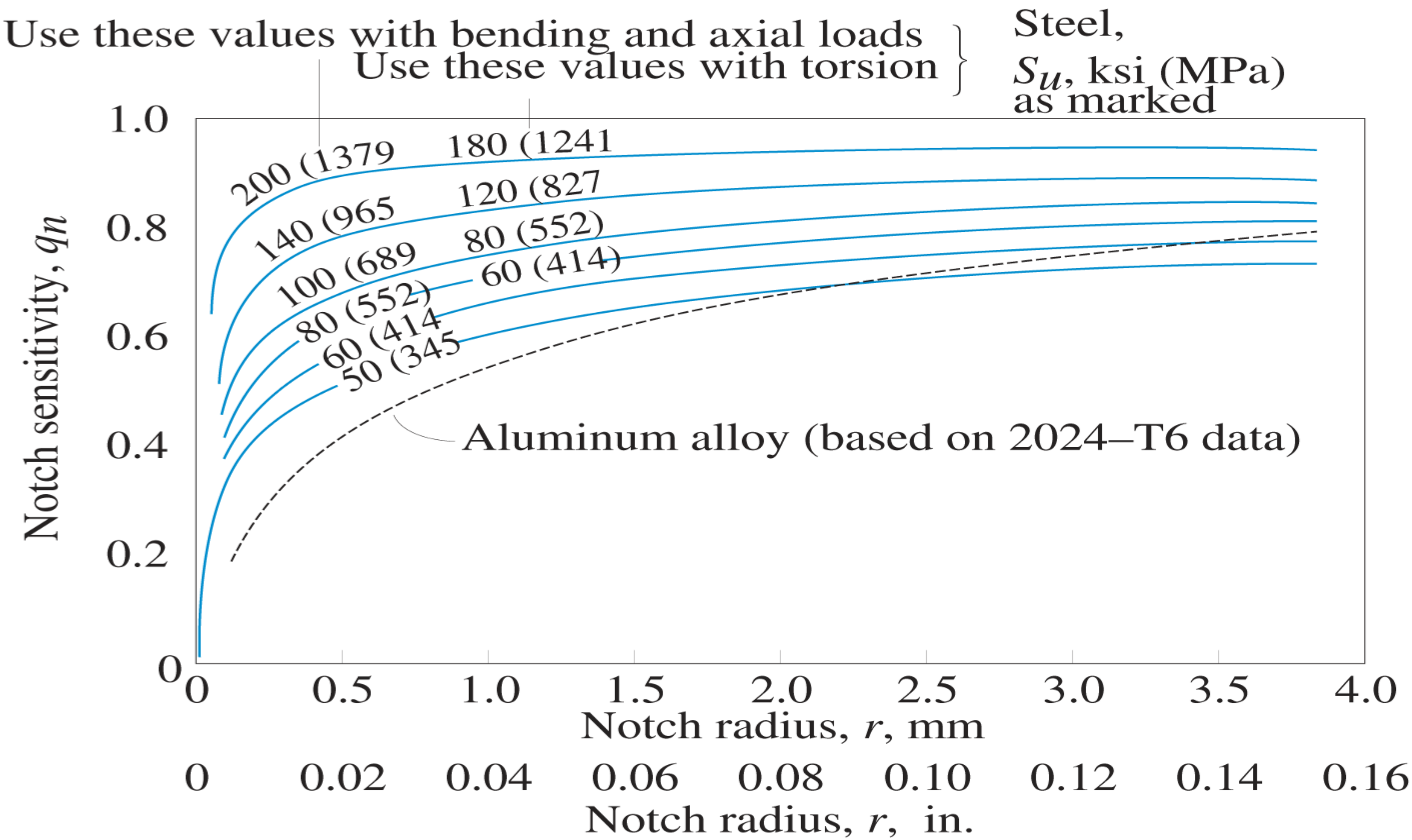
Notch Sensitivity Factors (Bending Example)



Notch Sensitivity Factors (Torsion Example)



Notch Sensitivity



Fatigue Stress Concentration Factors

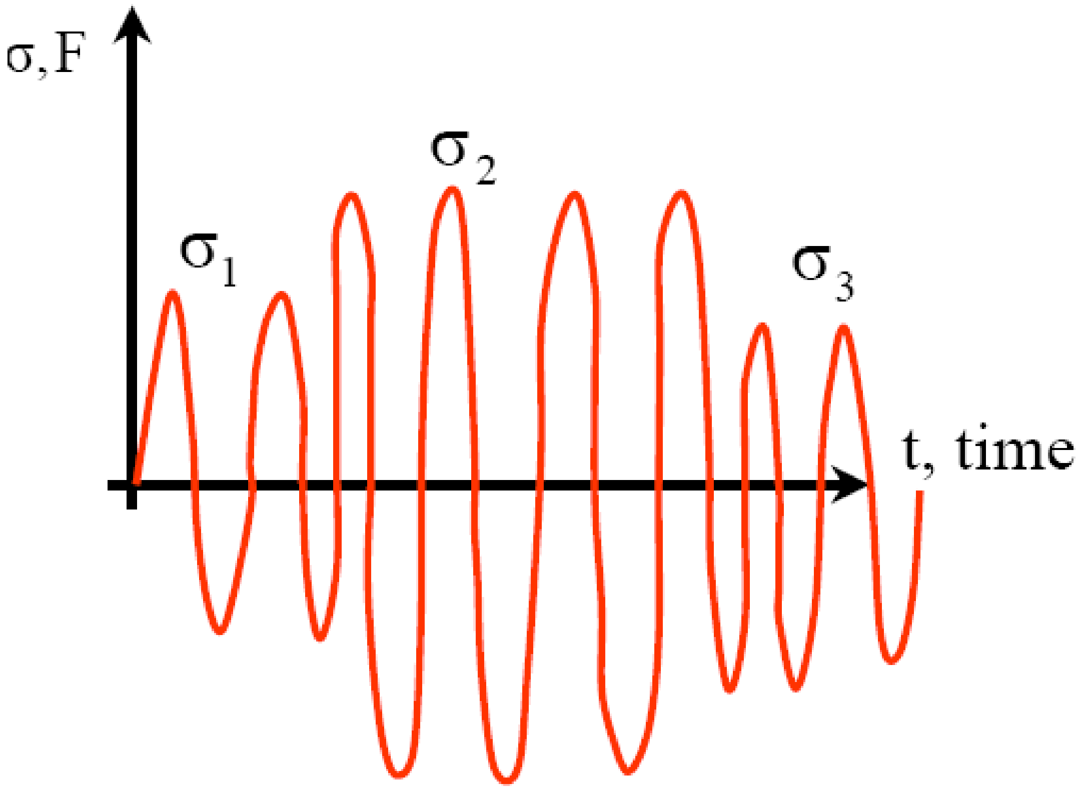
- K_f is normally used in fatigue calculations but is sometimes used with static stresses.

- Convenient to think of K_f as a stress concentration factor reduced from K_t because of lessened sensitivity to notches.

- If notch sensitivity data is not available, it is conservative to use K_t in fatigue calculations.

See Example!

Complex Loads



A part is subjected to completely reversed stresses as follows

σ_1 for n_1 cycles,

σ_2 for n_2 cycles,

σ_3 for n_3 cycles,

⋮

σ_m for n_m cycles,

What is the cumulative effect of these different load cycles?

Minor's Rule

Cumulative Damage Law

$$\frac{n_1}{N_1}+\frac{n_2}{N_2}+\frac{n_3}{N_3}+\ldots+\frac{n_m}{N_m}=C$$

$n_i \equiv$ number of cycles for stress level i

$N_i \equiv$ cycles to failure at stress level i

$C \equiv$ Constant ranging from 0.7 to 2.2.

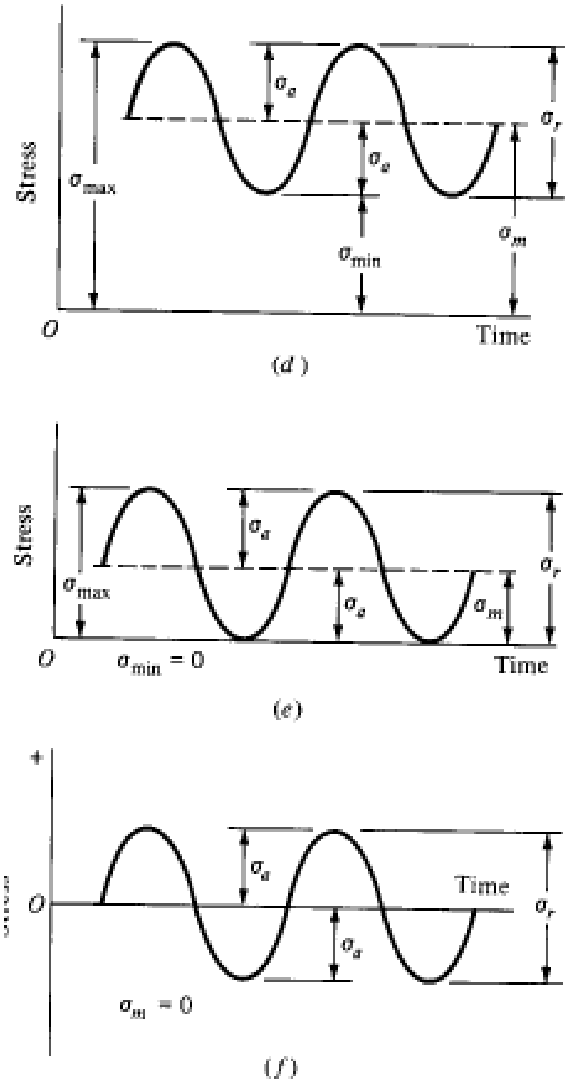
C is usually taken as 1.0

Minor's Rule is the simplest and most widely used Cumulative Damage Law

Example

- **Please write!**

Mean Stress Effects

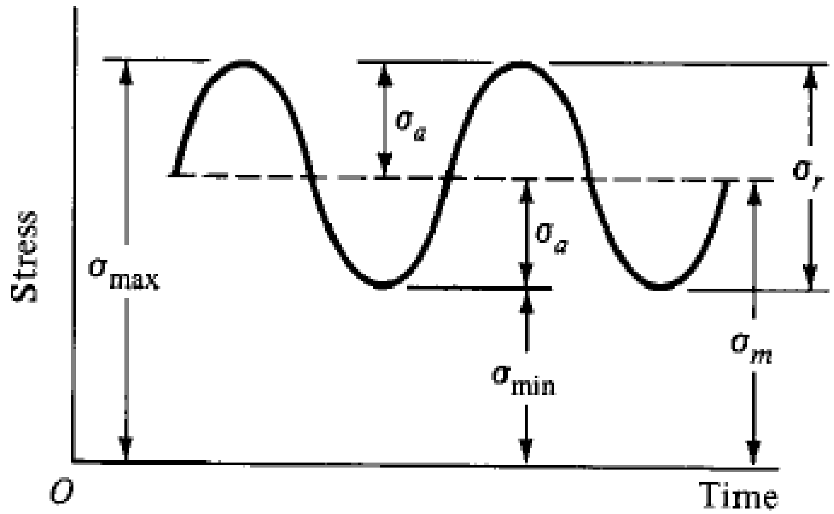


- The S-N curve obtained from a rotating beam test has completely reversed stress states.

- Many stress histories will not have completely reversed stress states.

Shigley, Fig. 7-12

Definitions



Stress Range

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Stress Ratio

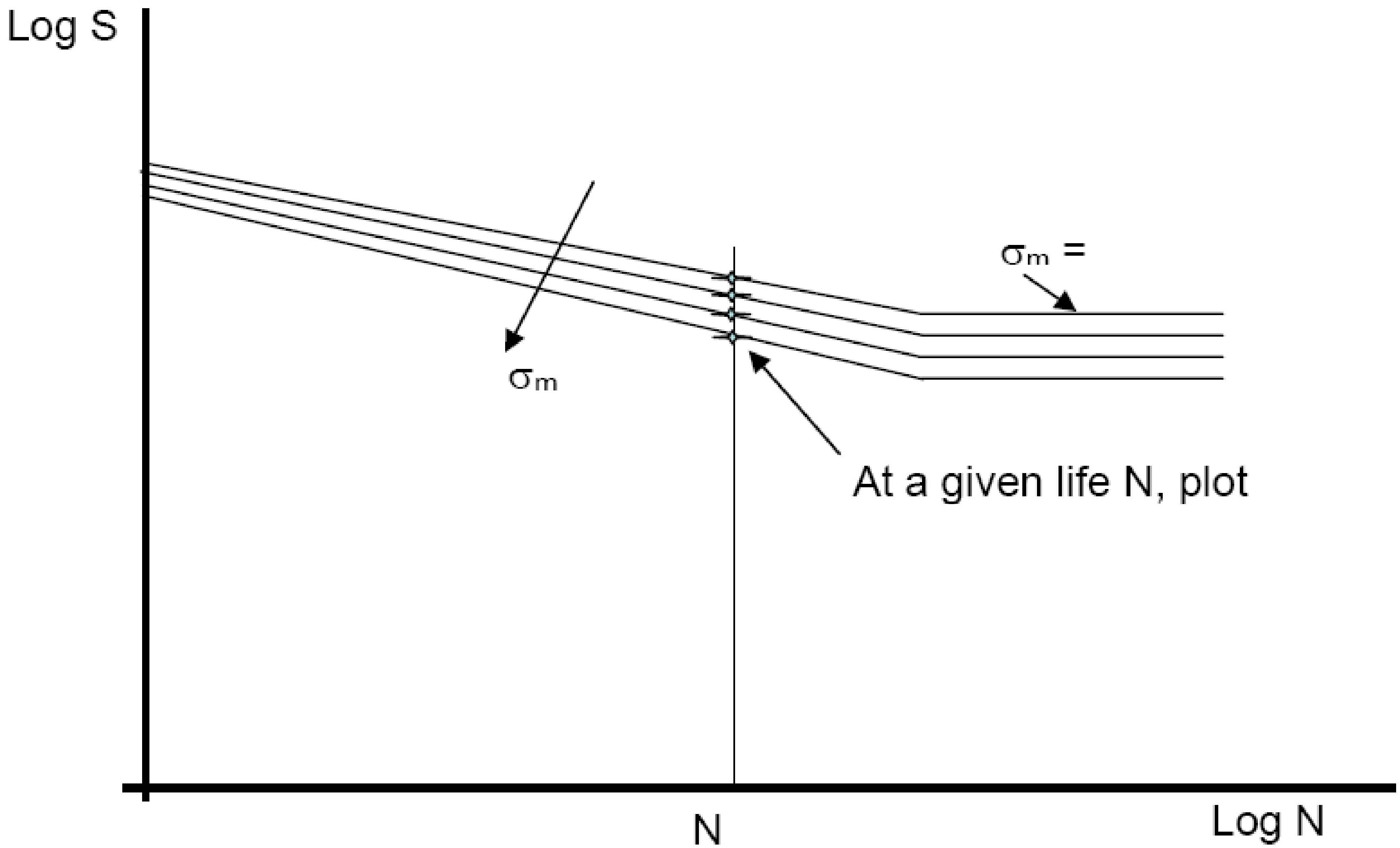
$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude Ratio

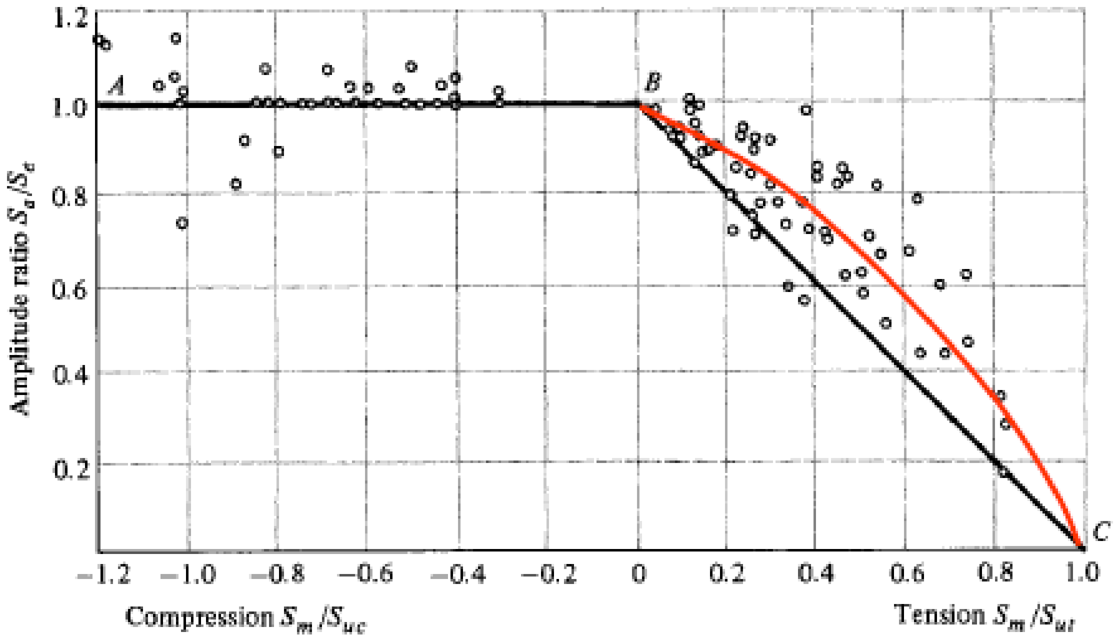
$$A = \frac{\sigma_a}{\sigma_m}$$

Note that R=-1 for a completely reversed stress state with zero mean stress.

Effect of adding a mean stress σmon the median S-N curve



Fluctuating Stress Failure Data



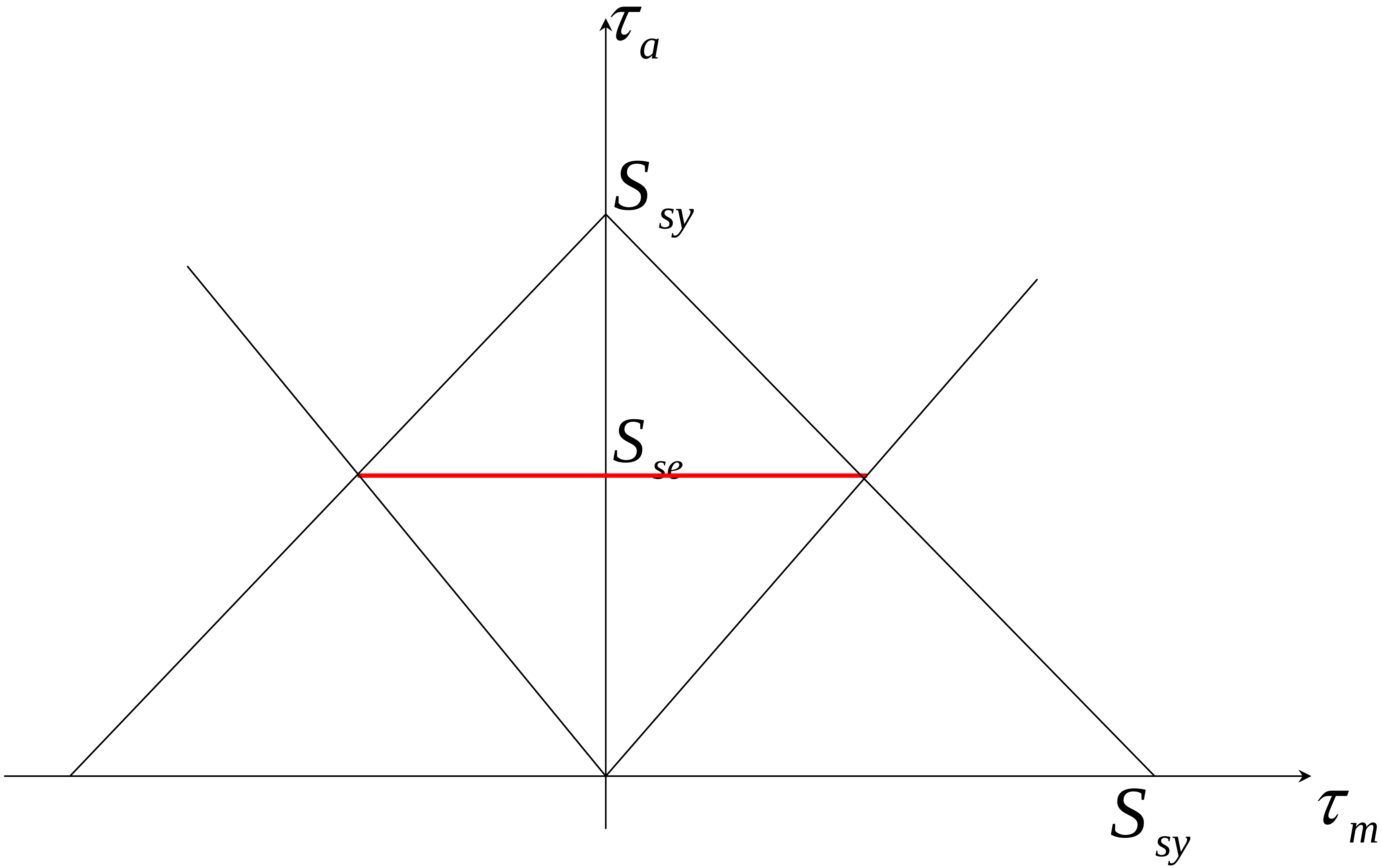
This plot shows the fatigue strength of several steels as a function of mean stress for a constant number of cycles to failure.

Note that a tensile mean stress results in a significantly lower fatigue strength for a given number of cycles to failure.

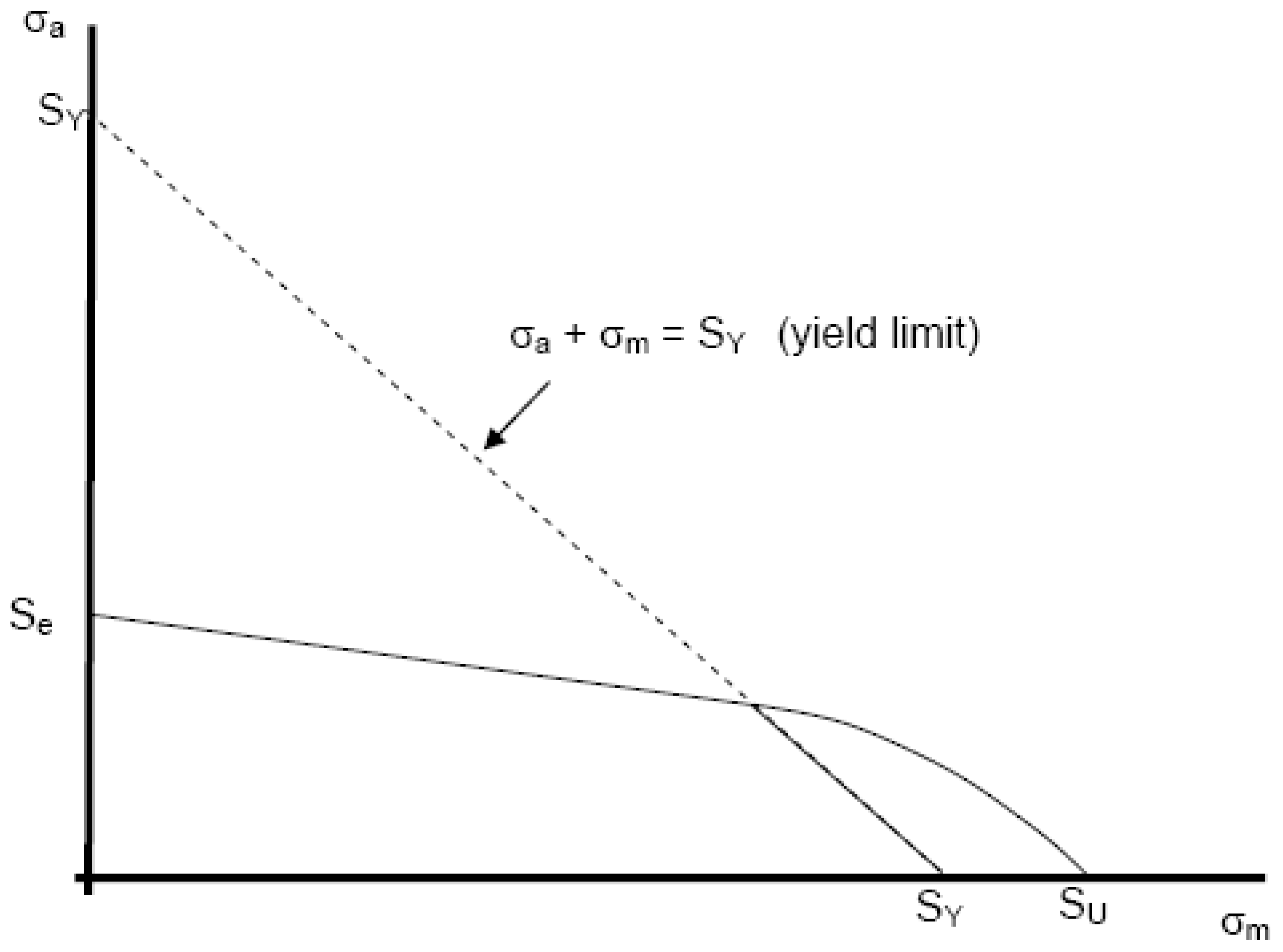
Note that a curved line passes through the mean of the data.

Shigley, Fig. 7-14

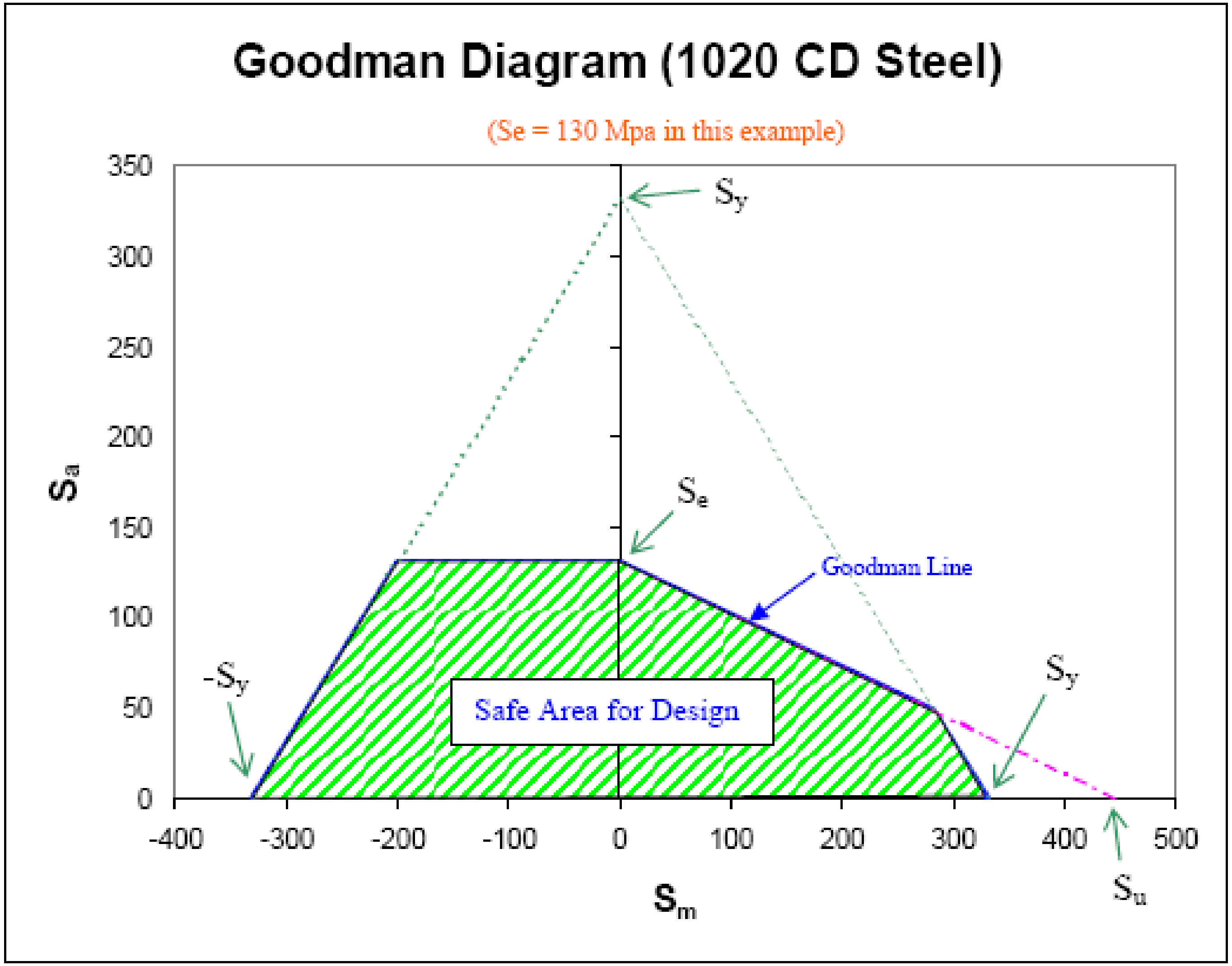
In Shear



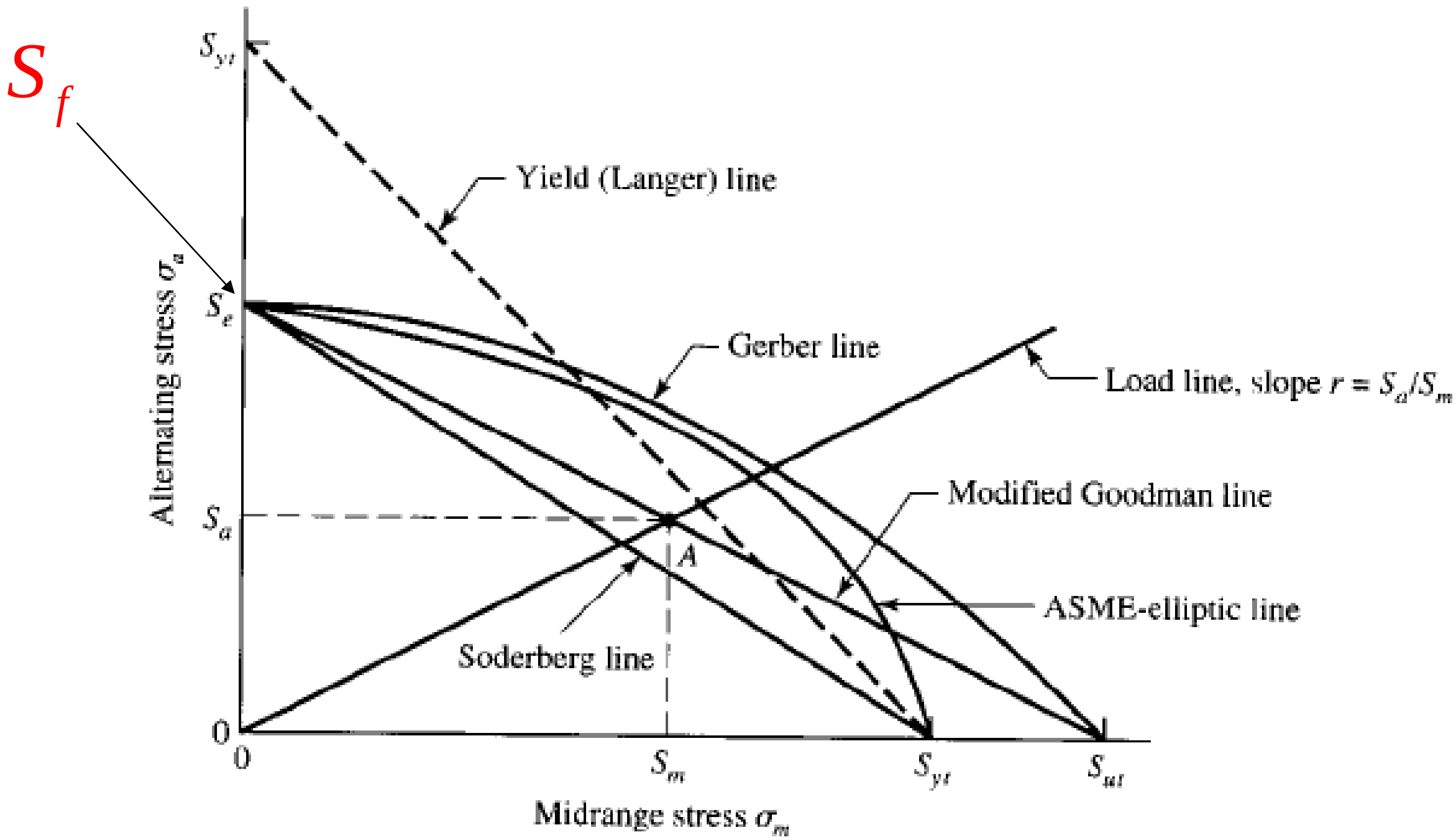
Effect of adding a mean stress σm on the median S-N curve



Effect of adding a mean stress σm on the median S-N curve



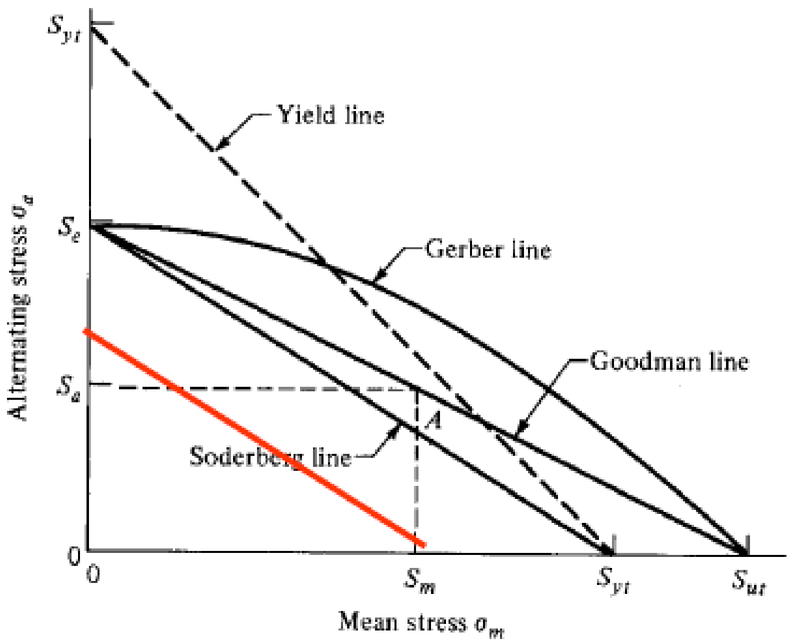
Fluctuating Stress Failure Interaction Curves



Soderberg Interaction Line

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$

Any combination of mean and alternating stress that lies on or below the Solderberg line will have infinite life.



Factor of Safety Format

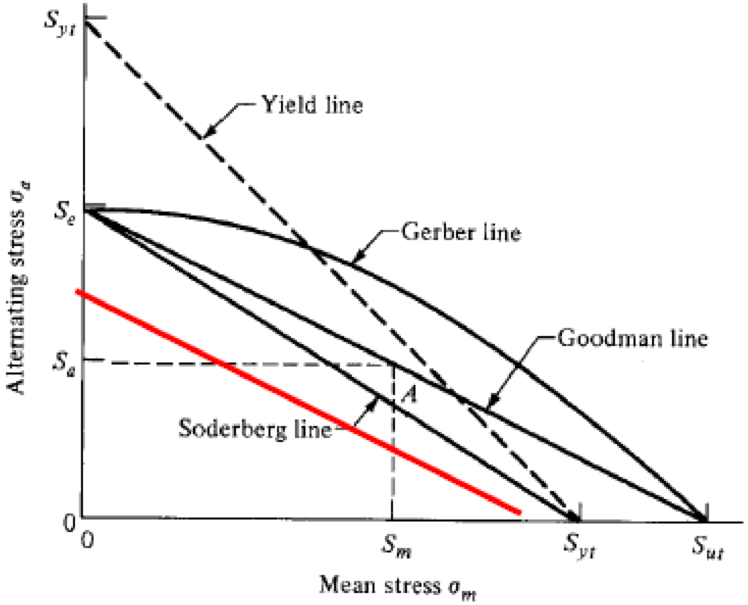
$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{yt}} = \frac{1}{N_f}$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

Goodman Interaction Line

$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Any combination of mean and alternating stress that lies on or below the Goodman line will have infinite life.



Factor of Safety Format

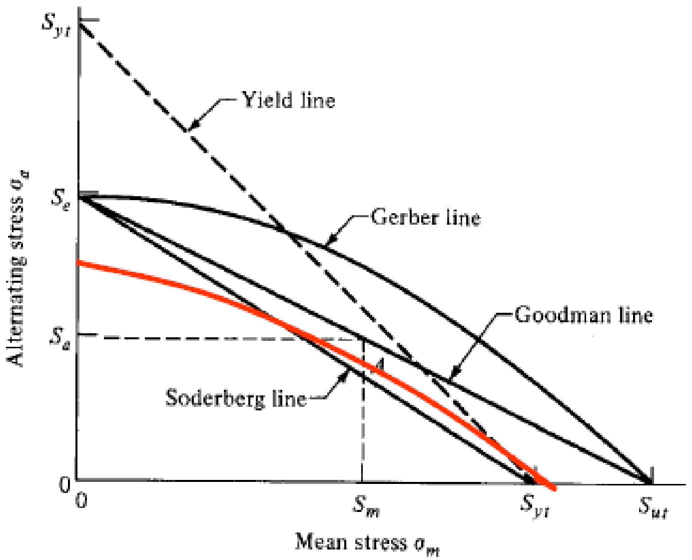
$$\frac{k_f S_a}{S_e} + \frac{S_m}{S_{ut}} = \frac{1}{N_f}$$

Note that the fatigue stress concentration factor is applied only to the alternating component.

Gerber Interaction Line

$$\frac{k_f S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Any combination of mean and alternating stress that lies on or below the Gerber line will have infinite life.

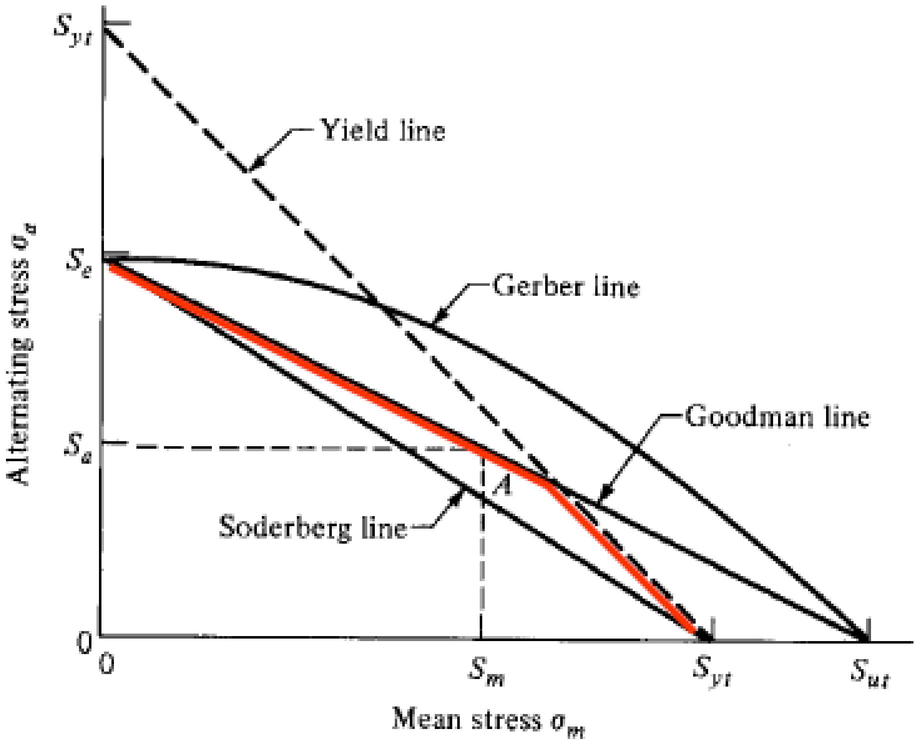


Factor of Safety Format

$$\frac{k_f N_f S_a}{S_e} + \left(\frac{N_f S_m}{S_{ut}}\right)^2 = 1$$

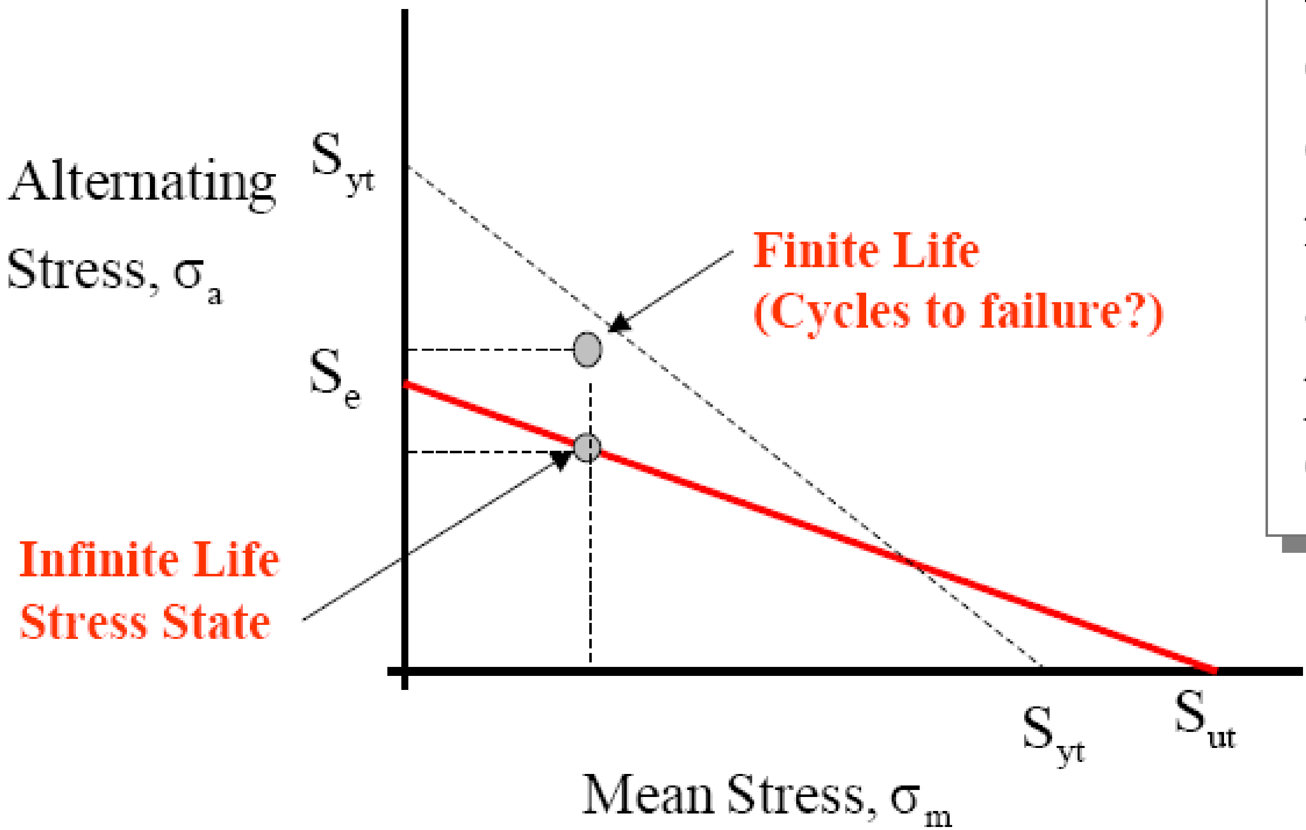
Note that the fatigue stress concentration factor is applied only to the alternating component.

Modified-Goodman Interaction Line



The Modified-Goodman Interaction Line never exceeds the yield line.

Finite Life Estimates



How can the life of a part be estimated if the mean stress-alternating stress pair lie above the Goodman line?

Goodman Diagram

Torsional Fatigue strength

In constructing the Goodman diagram, Joerres uses

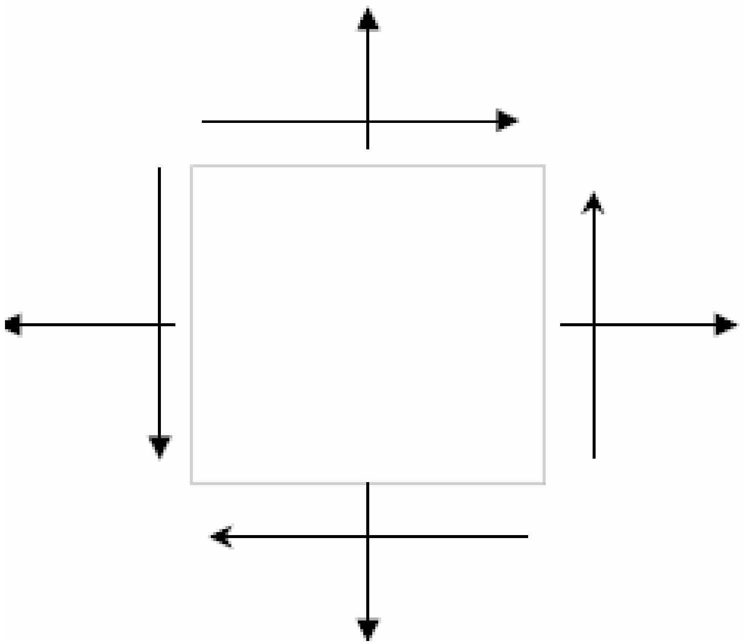
$$S_{su} = 0.67 S_{ut} \tag{7–56}$$

Also, from Chap. 6, $S_{sy} = 0.577 S_{yt}$ from distortion-energy theory, and the mean load factor k_c is given by Eq. (7–25), or 0.577. This is discussed further in Chap. 10.

Multi-axis Fluctuating Stress States

Everything presented on fatigue has been based on experiments involving a single stress component.

What do you do for problems in which there are more than one stress component?



Multi-axis Fluctuating Stress States

- **Assuming that all stress components are completely reversing and are always in time phase with each other:**

1. For the strength, use the fully corrected endurance limit for bending, S_e.
2. Apply the appropriate fatigue stress-concentration factors to the torsional stress, the bending stress, and the axial stress components.
3. Multiply any alternating axial stress components by the factor $1/k_{c,\mathrm{ax}}$.
4. Enter the resultant stresses into a Mohr's circle analysis and find the principal stresses.
5. Using the results of step 4, find the von Mises alternating stress σ'_a.
6. Compare σ'_a with S_a to find the factor of safety.

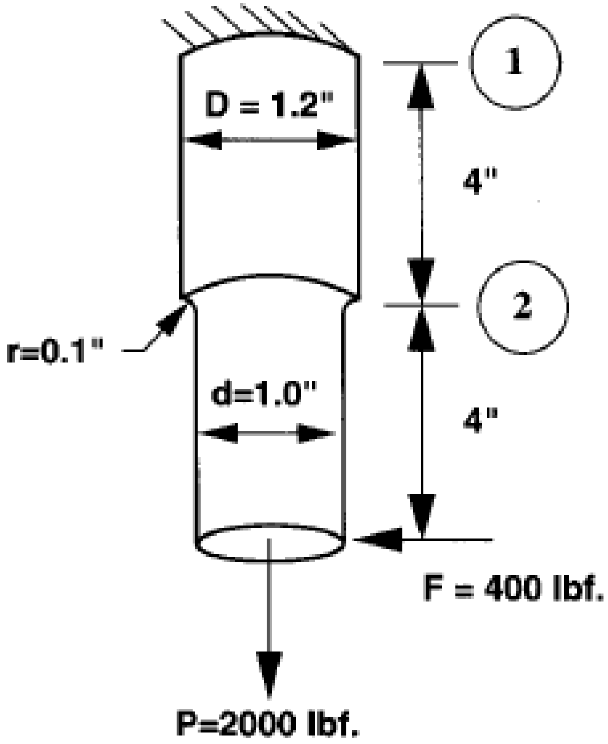
Multi-axis Fluctuating Stress States

If the stress components are not in phase but have the same frequency, the maxima can be found by expressing each component in trigonometric terms, using phase angles, and then finding the sum. If two or more stress components have differing frequencies, the problem is difficult; one solution is to assume that the two (or more) components often reach an in-phase condition, so that their magnitudes are additive.

If midrange stresses are also present, then steps 4 and 5 can be repeated for them and the resulting steady von Mises stress component σ'_m used with σ'_a in forming a Gerber or ASME-elliptic solution. Both the steady and amplitude components are augmented by K_f or K_{fs} stress-concentration factor.

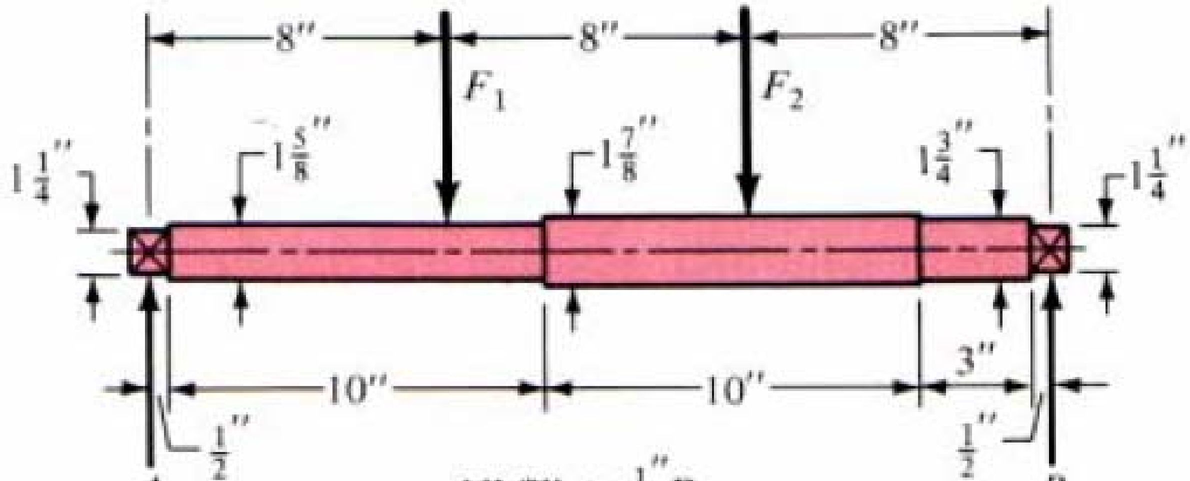
Assignment

- Find the most critically stressed location on the stepped shaft. Note that you will need to use the stress concentration factors contained in the lecture notes.



Assignment

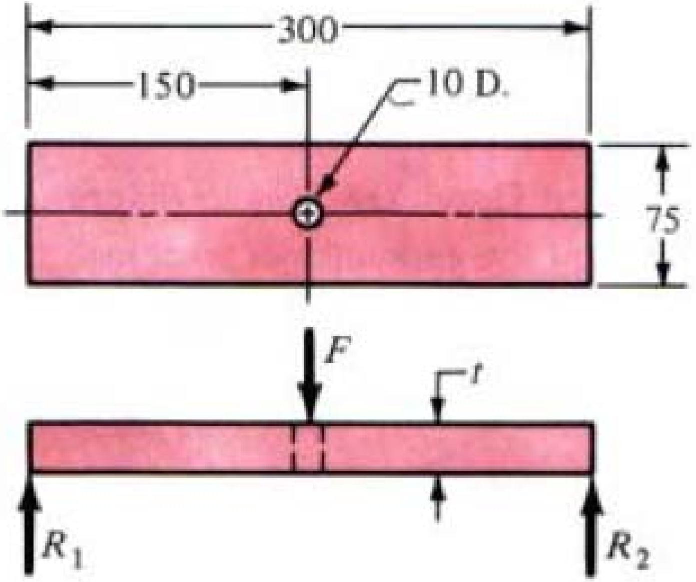
The steel shaft shown in the figure has a ground finish and a minimum tensile strength of 89 kpsi. The shaft rotates at 1720 rev/min and is supported in rolling bearings at A and B. Estimate the life of the part if the forces are F_1 = 2000 lb and F_2 = 3000 lb.



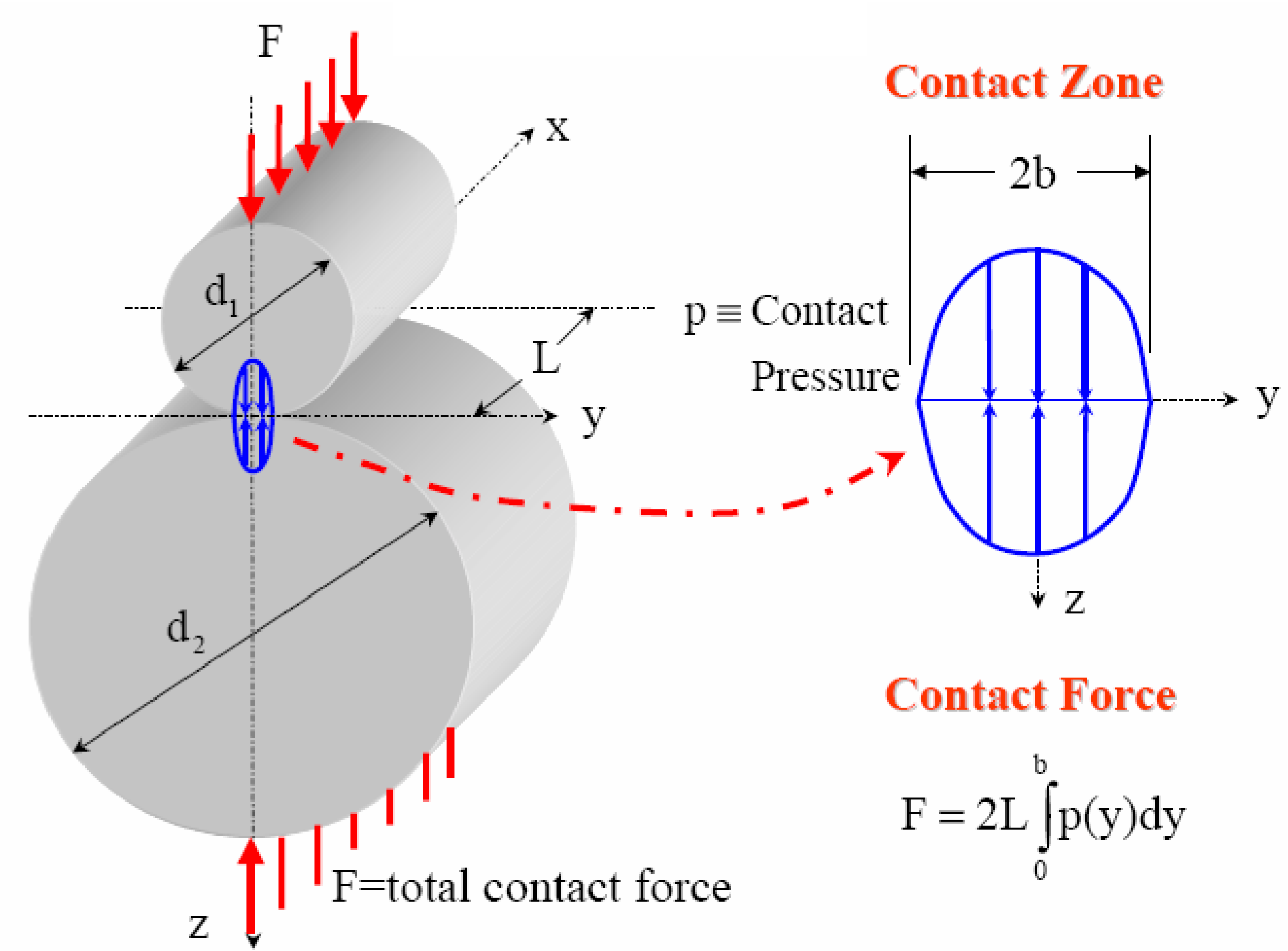
All fillets $\frac{1}{16}''$ R.

Assignment

The flat steel spring illustrated is loaded in bending by the force F. The spring supports a static weight of exactly 9.36 kN. During operation, the total load on the spring is estimated to fluctuate up to 10.67 kN maximum. The spring is forged of a 95-point carbon steel and after heat treatment has the following minimum properties: $S_{ut} = 1400$ MPa, $S_{yt} = 950$ MPa, $H_B = 399$, and 32 percent reduction in area. Estimate the factor of safety if the spring is 18 mm thick.



Contact Stress Between Two Cylinders

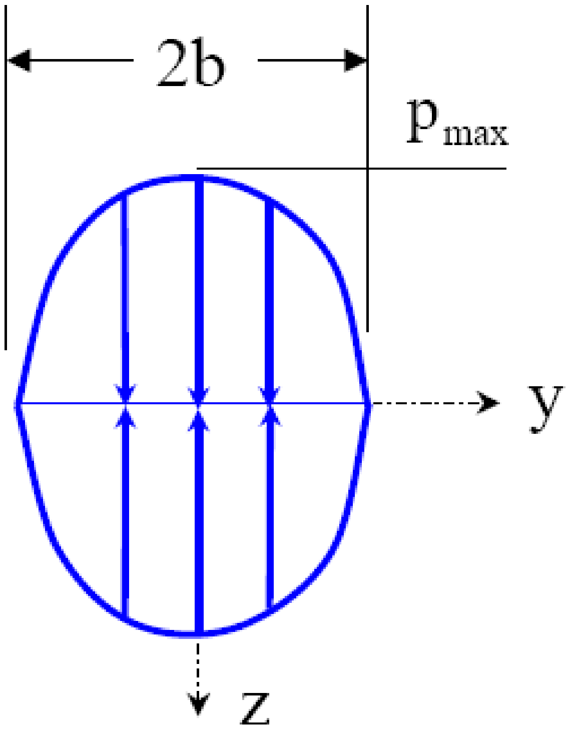


Contact Force

$$F = 2L\int_0^b p(y)dy$$

Hertz Contact Stress Equations

Contact Zone



Contact Width

$$b=\sqrt{\frac{2F}{\pi L}\frac{\left(1-\nu_1^2\right)/E_1+\left(1-\nu_2^2\right)/E_2}{1/d_1+1/d_2}}$$

Maximum Contact Pressure

$$p_{max}=\frac{2F}{\pi bL}$$

Stress components

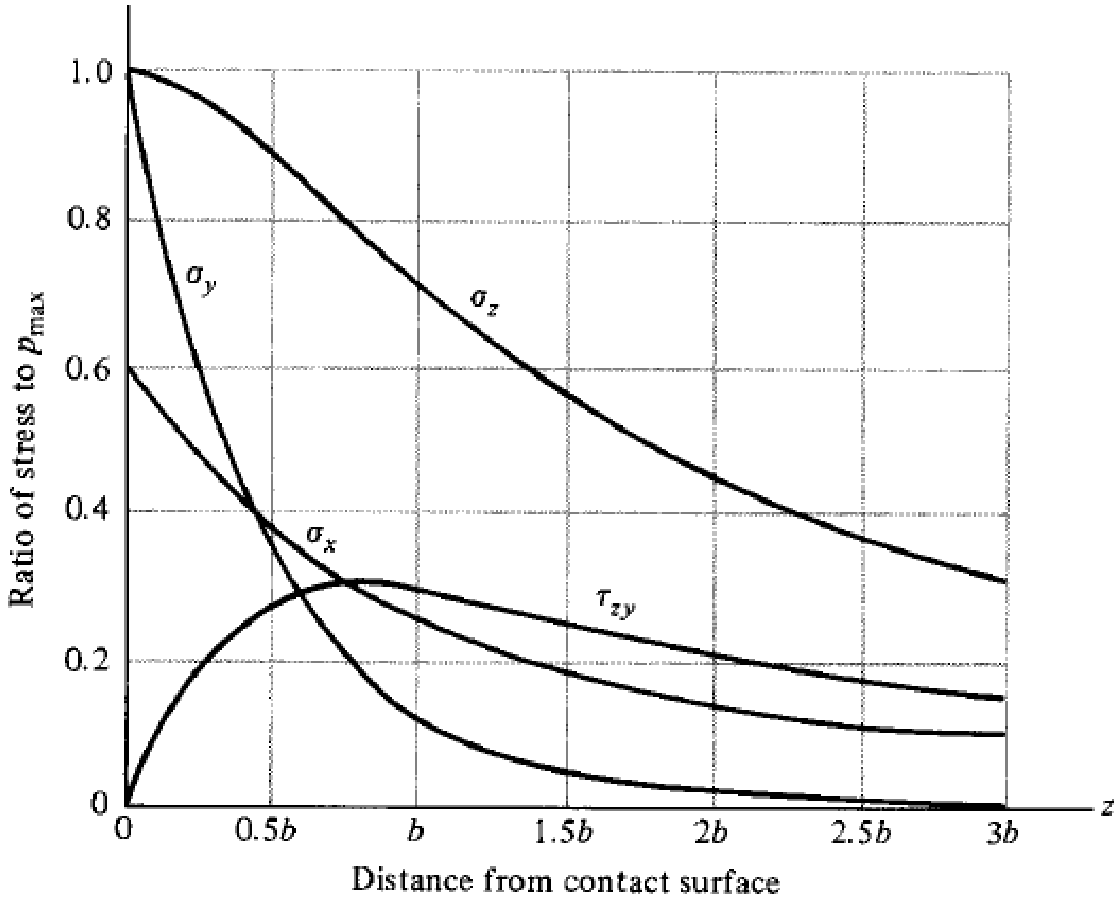
The stress state along the z axis is given by the equations

$$\sigma_x = -2\nu p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$

$$\sigma_y = -p_{\max}\left(\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right)$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{\sqrt{1+z^2/b^2}}$$

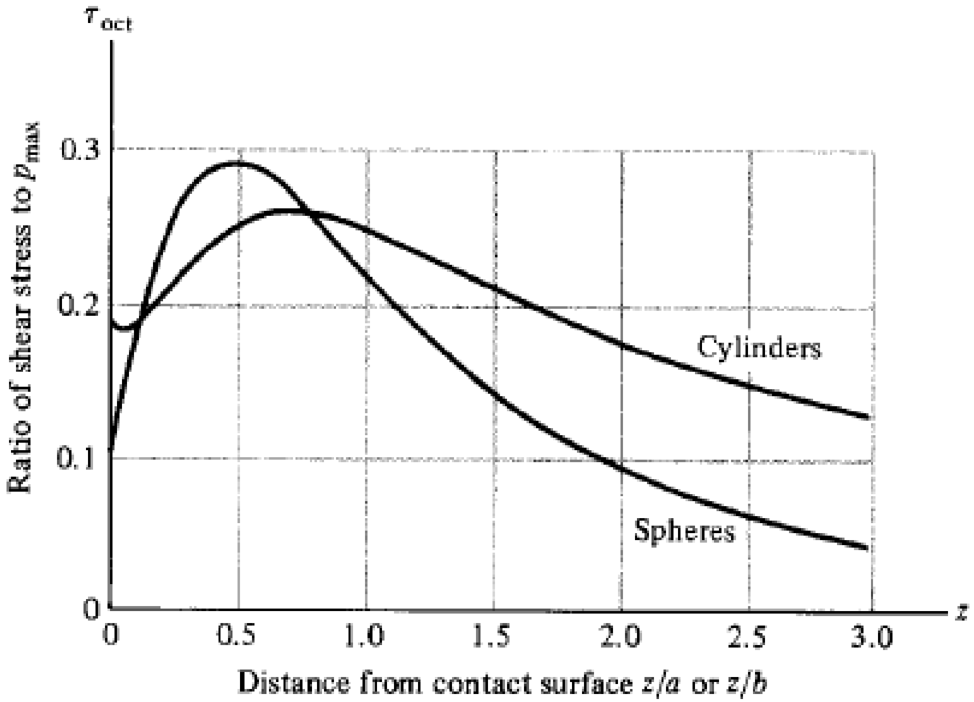
Hertz Contact Stress Equations



This graph shows the variation of the stress components along the z-axis.

Note that the maximum shear stress is much less than the maximum contact pressure.

Hertz Contact Stress Equations



Von Mises stress variation along the z-axis.

Note that the von Mises stress is much less than the maximum contact pressure.

$$\sigma_{eff} = 0.26 \cdot p_{max}$$

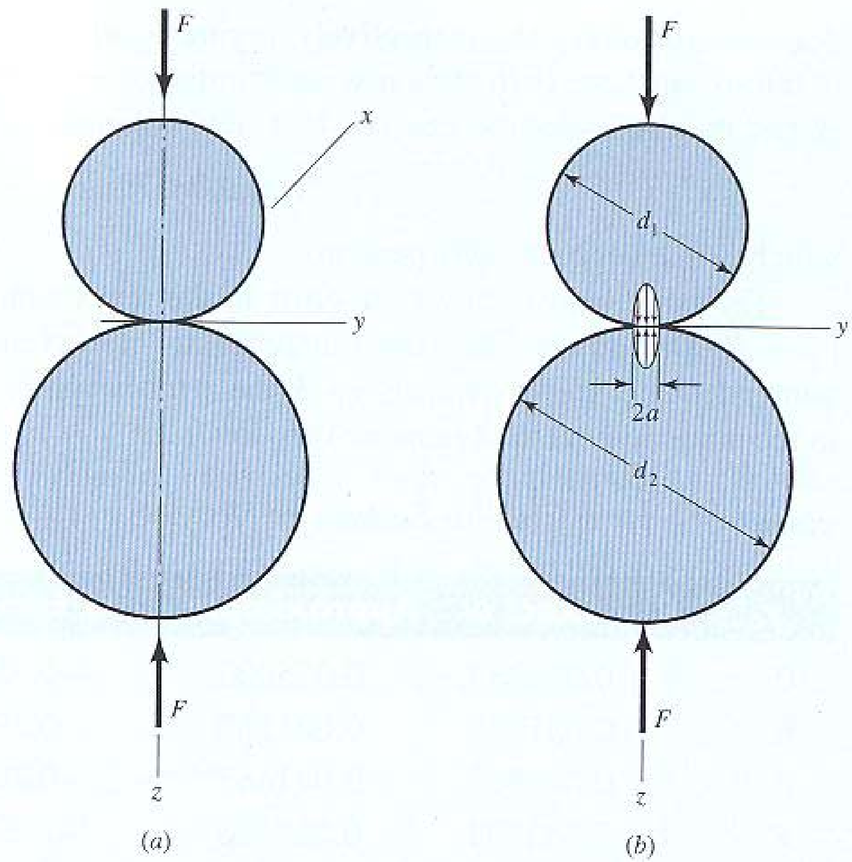
$$\frac{S_{yt}}{N_{fs}} = 0.26 \cdot p_{max}$$

$$\text{if } N_{fs} = 1.0$$

$$\text{then allowable } p_{max} \approx 3.85 \cdot S_{yt}$$

Contact Stress Between Two spheres

(a) Two spheres held in contact by force F; (b) contact stress has a hemispherical distribution across contact zone diameter 2a.



$$a = \sqrt[3]{\frac{3F}{8}\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$p_{\max} = \frac{3F}{2\pi a^2}$$

Contact Stress Between Two spheres

The maximum stresses occur on the z axis, and these are principal stresses. Their values are

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\dfrac{z^2}{a^2}\right)}\right] \tag{4–74}$$

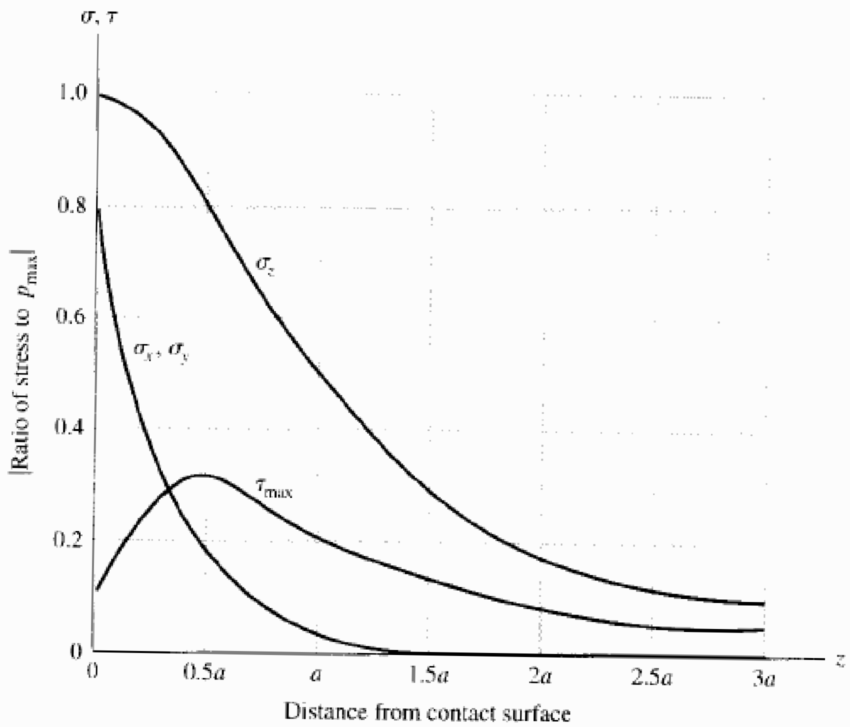
$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1+\dfrac{z^2}{a^2}} \tag{4–75}$$

These equations are valid for either sphere, but the value used for Poisson's ratio must correspond with the sphere under consideration. The equations are even more com-

Contact Stress Between Two spheres

Mohr's circles for the stress state described by Eqs. (4–74) and (4–75) are a point and two coincident circles. Since $\sigma_1 = \sigma_2$, we have $\tau_{1/2} = 0$ and

$$\tau_{max} = \tau_{1/3} = \tau_{2/3} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_2 - \sigma_3}{2} \qquad (4\text{–}76)$$



Hertzian Endurance Strength

$$S_c = P_{\max}$$

$$S_c = 2.76 H_B - 70 \ \ MPa$$